







Digitized by the Internet Archive  
in 2019 with funding from  
University of Alberta Libraries

<https://archive.org/details/3000psihydraulic00dona>





Thesis  
1965  
Page 2

THE UNIVERSITY OF ALBERTA

A 3000 PSI HYDRAULIC POSITION CONTROL SYSTEM

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES  
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE  
OF MASTER OF SCIENCE

DEPARTMENT OF ELECTRICAL ENGINEERING

By

DONALD HENRY ALBERT PIROT

EDMONTON, ALBERTA

MAY, 1965



UNIVERSITY OF ALBERTA  
FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read,  
and recommend to the Faculty of Graduate Studies for  
acceptance, a thesis entitled, A 3000 PSI Hydraulic Position  
Control System, submitted by Donald Henry Albert Pirot  
in partial fulfilment of the requirements for the degree  
of Master of Science.

---





## ABSTRACT

This report describes the development, analysis, and testing of a hydraulic position control system designed to operate at 3000 p.s.i. The system controls the linear position of a load that can be varied in 20 lb. steps from 20 lbs. to 100 lbs. A variable capacitance position transducer is used to sense the actuator position.

The system was tested to obtain its open and closed loop frequency response and its transient response with various loop gains. It was also operated as a relay-controlled system and the transient response in this mode was compared with the linear response.

The system was found to be linear over the range of operating conditions used. The open loop system acted as a pure integrator yielding an inherently overdamped closed loop transient response. The relay-controlled system was found to be potentially as fast responding as the linear system.



## ACKNOWLEDGEMENTS

The writer wishes to express his appreciation for the encouragement and assistance received during the preparation of this work. The research described in this thesis was carried out at the Department of Electrical Engineering, University of Alberta, under the supervision of Professor Y. J. Kingma, to whom the writer wishes to acknowledge his indebtedness for advice and assistance throughout the work.

The writer wishes to thank other members of the Department for their cooperation and helpful suggestions.

The technical assistance of the Electrical Engineering Machine Shop Staff is gratefully acknowledged.

Financial assistance from the National Research Council is gratefully acknowledged.



## TABLE OF CONTENTS

	Page No.
I. INTRODUCTION	
1.1 Purpose	1
1.2 Advantages of Hydraulic Control	2
1.3 Disadvantages of Hydraulic Control	4
1.4 Summary	7
II. DESIGN	
2.1 Design Principle	8
2.2 System Development	8
2.3 The Electrohydraulic Valve	9
2.4 The Linear Actuator	12
2.5 Design of a Suitable Position Transducer	13
2.6 Variable Capacitance Transducer	15
2.7 Amplifiers	17
2.8 Power Supplies	18
2.9 Actuator Load	18
III. ANALYSIS OF LINEAR OPERATION	
3.1 General	20
3.2 Block Diagram	21
3.3 The Orifice Equation	22
3.4 Amplifiers	24
3.5 Torque Motor	25
3.6 Balanced Flapper Valve	29
3.7 The Spool Valve	31
3.8 Position Transducer	34
3.9 Transfer Function of the Torque Motor	36
3.10 Transfer Function of the Spool Valve	37





3.11	Transfer Function of the Actuator	38
3.12	Effect of Compressible Hydraulic Fluid	41
3.13	Resonance of Structure	42
3.14	Summary	43
3.15	Nonlinear Effects	43
IV.	DISCONTINUOUS OPERATION	
4.1	Principle	45
4.2	Advantages of Relay Operation	45
4.3	Disadvantages of Relay Operation	46
4.4	Analysis of Operation	46
4.5	Describing Function Method	47
4.6	Phase-Plane Method	48
4.7	Simulation of a Two Position Relay	49
V.	EXPERIMENTAL TESTS AND RESULTS	
5.1	General	50
5.2	Open Loop Measurements - Linear Operation	50
5.2	Closed Loop Measurements - Linear Operation	52
5.3	Transient Response - Linear Operation	52
5.4	Measurements on Relay System	53
5.6	Open Loop Results - Linear Operation	56
5.7	Closed Loop Results - Linear Operation	59
5.8	Transient Results - Linear Operation	59
5.9	Relay Operation - Results	61
VI.	CONCLUSIONS	63
VII.	RECOMMENDATIONS	64



BIBLIOGRAPHY	66
APPENDIX A	67
APPENDIX B	72
APPENDIX C	78

FIGURES	After Page 78
---------	---------------



## LIST OF FIGURES

- Figure 1 - Block Diagram of the Electrohydraulic System.
- Figure 2 - Load Flow - Load Pressure Curve for Moog 71-101 Electrohydraulic Valve
- Figure 3 - Flow Gain Curve of Moog 71-101 Valve
- Figure 4 - Schematic of Moog 71-101 Valve
- Figure 5 - Detail of Torque Motor and Flapper Valve
- Figure 6 - Detail of Spool Valve
- Figure 7 - System Load
- Figure 8 - Capacitance and Control Characteristics of Position Transducer
- Figure 9 - a) Section of Position Sensing Element  
b) Diode - Capacitor Bridge of Position Transducer
- Figure 10- Position Transducer  
a) Equivalent Circuit for Positive Half Cycle  
b) Equivalent Circuit for Negative Half Cycle  
c) Waveform for Capacitor  $C_1$   
d) Waveform for Capacitor  $C_2$
- Figure 11- Frequency Response Curves for Moog 71-101 Valve
- Figure 12- a) Block Diagram of Relay - Controlled Servo  
b) Phase Plane Curve of Relay -Controlled Servo
- Figure 13- Schematic of Circuit used to Simulate 2 Position Relay
- Figure 14- a) Testing Circuit - Linear Operation  
b) Testing Circuit - Relay Operation
- Figure 15 - Block Diagram of High Pressure Hydraulic Power Supply
- Figure 16 - Open Loop Frequency Response Curve





Figure 17 - Closed Loop Frequency Response Curve

Figure 18 - Nichols Diagram of Closed Loop Response Curve

Figure 19 - Closed Loop Response - System with Additional  
Gain of 5.

Figure 20 - Closed Loop Response - System with Additional  
Gain of 10.

Figures 21 to 23 Inclusive - Transient Response - Linear  
System.

Figures 24 and 25 - Transient Response - Relay System.



## CHAPTER I

### 1.1 Purpose

The immediate purpose of this project was the building and testing of a hydraulic motor to operate at 3000 p.s.i.

The project consisted of three phases. The first phase, the construction of a high pressure power supply was merely a matter of assembling a number of commercially available components, taking into consideration convenience of operation and safety.

The second phase involved assembling the hydraulic motor and devising a means of testing the system response under various load conditions. The former necessitated the development of a position transducer capable of measuring the displacement of the actuator to a considerable degree of accuracy over a piston stroke of ten inches and the latter, the building of a wheeled platform upon which could be placed 20 pound lead ingots to a maximum load of 100 pounds.

The reason for wishing to vary the load on the system is that hydraulic motors are inherently non-linear. Their dynamic characteristics are a function of the system load and the amplitude of the input signal. Thus it is desirable to be able to vary both of these quantities.

The final phase of the project was the evaluation of system dynamic characteristics using frequency response methods, and the operation of the system as a relay servo and analysis of this non-linear operation using standard techniques of non-linear analysis.



A more far-reaching goal of this project was to lay the groundwork for the eventual development of a machine tool control system. Both linear and discontinuous operation of servovalves have been investigated in order to accumulate evidence in favour of either type of operation. Discontinuous control has obvious advantages in its simplicity and lower cost. An investigation was made to find out if it would be possible to achieve, with this type of control, a speed of response and an accuracy comparable to that of linearly controlled systems.

## 1.2 Advantages of Hydraulic Control

When considering various types of power that could be used in machine tool control, hydraulic devices must be compared with electromechanical and pneumatic devices. Listed here are several important ways in which hydraulic power has an advantage over other types of power.

A) High power output - At high pressure operation a hydraulic device can deliver a large amount of power. The amount of power delivered depends on the flow capacity of the device and on the operating pressure which in turn depends on the strength of the materials of which the device is constructed. At a load pressure of 2000 p.s.i. with a flow of 3.75 gallons (U.S.)/minute (conditions for maximum power output in the type 71-101 Moog valve) the power output would be 4.36 horsepower.





B) High power to weight and power to volume ratios - These ratios in electromechanical devices are limited by the saturability of magnetic core materials. In hydraulic devices, the only limiting factor is the strength of the materials used.

C) High rigidity - The low compressibility of hydraulic fluids yields a highly rigid system, compared to highly compressible (and therefore springy) pneumatic systems and electromechanical devices in which there is some compliance because the torque due to magnetic fields is not enough to hold an armature completely rigid.

D) Fast response - Because of the large forces found in hydraulic devices, high accelerations are possible. The low compressibility of hydraulic fluid also helps to make possible fast response.

E) High power gains - In a multi-stage servovalve, because flow in the output stage is perpendicular to the direction of spool motion, the only forces required to position the spool are those needed to overcome the usually small spool friction, to compress the springs which centre the spool, and to overcome the small feedback pressure due to unsymmetrical pressure distribution in the spool chambers. Thus a large amount of power can be controlled with a very small input power. In the Moog type 71 valve, operating with a supply pressure of 3000 p.s.i., 4.36 horsepower can be delivered with an input power of 45 milliwatts.

F) Hydraulic components are compatible with other types of devices. Electrohydraulic transducers are common.



It is possible to obtain commercially many types of pneumatic-hydraulic transducers. This does not necessarily give hydraulics an advantage over other types of power but hydraulic power is certainly at par in this aspect.

G) Hydraulic devices are not as sensitive to shock and vibration as are electromechanical devices.

### 1.3 Disadvantages of Hydraulic Control

#### A) Non-linearity -

1) Inherent non-linearity. This type of non-linearity may not be significant for small-signal and small-load operation. However, when loads are large the pressure difference required to accelerate the load may become an appreciable fraction of the supply pressure. This reduces the valve pressure drop and effectively lowers the saturation flow of the device. This type of non-linearity is discussed further in section 3.15. It is very significant because if the valve is to operate efficiently, the load pressure drop should be about two thirds of the total supply pressure (for maximum power transfer).

#### 2) Non-linearities due to valve construction.

a) Non-linear characteristic of the torque motor. This is not a discontinuous non-linearity and in a well designed valve is insignificant over the normal operating range of the device.

b) Overlap of spool in a closed centre valve



causes dead zone. Underlap of the spool causes the null gain of the valve to be greater than the gain at higher flows.

c) Saturation - Because of the finite dimensions of the orifices and the limit on the volume of fluid that can be drawn from the power supply, saturation occurs at large input signals.

3) Other non-linearities - 'Stiction', backlash and hysteresis are found in valves, actuators, and position transducers.

B) Hydraulic systems can be messy. It is necessary to employ special methods to eliminate leaks and minimize spills. Special sealants are used as well as rubber O-rings and packings of various types.

C) A clean supply of hydraulic fluid is essential and infinite care must be taken to keep small particles of metal and grit out of the system. Because of the precise tolerances used in machining servovalves for linear operation a particle as small as several microns in diameter is capable of scoring the spool and changing the flow characteristics of the valve quite markedly. Before starting a hydraulic system for the first time it may be necessary to flush the system completely. High filtration is not necessary in a discontinuous system where the machining tolerances are not as high due to the on-off nature of operation. This partly accounts for the lower cost of this type of operation.







D) Because of the unsymmetrical construction of most types of hydraulic devices, there is a tendency for vibration to be present. Where there are sudden surges and reversals of flow, shock waves may develop. In general, hydraulic devices are not as smooth running and quiet as electromechanical devices.

E) There may be relatively high transmission losses and power dissipation in control orifices due to friction, especially when there is turbulent flow.

F) Temperature sensitivity - In some valves, the orifice coefficients are a function of the temperature of the hydraulic fluid. For valves in which turbulent flow is assured by the operating pressures and geometry of the system, such as the Moog valves used in this project, the valve characteristics are not affected by small temperature changes.

G) Cost - Because of the required machining tolerances in orifices and spools, proportional control valves are expensive. For this report an investigation was made into operation of the system as a relay servo. If it is possible to obtain high response speed and accuracy with this type of operation, much less expensive valves can be used in the system.

H) Inflexibility - Minimum bend radii in tubing and hose make a hydraulic system less flexible than an electrical system.

I) There is a minimum practical size for a hydraulic component. It is a function of operating pressure or the maximum force that the system is required to produce.



#### 1.4 Summary

In spite of the disadvantages, some of which are being overcome by more advanced manufacturing techniques and better methods of analysis, the high power outputs and high gains available with hydraulic systems make their use attractive in many applications and essential in some applications, particularly in the aircraft industry. It is hoped that this study will be of use in the development of a highly accurate and fast responding machine tool control device which may operate entirely on fluid power due to possible use of fluid logic devices in the numerical unit.



## CHAPTER II

DESIGN2.1 Design Principle

The usual method of design of a hydraulic system is to specify the closed loop response characteristics and desired accuracy and from these, to choose components which come closest to meeting a compromise between specifications and component costs. This method has not been used here because certain components were already available. It was decided to build the system around a type 71-101 Moog electro-hydraulic valve. The operation and characteristics of this valve are discussed in section 2.3.

2.2 System Development

The primary purpose of this report was to obtain data on the dynamic characteristics of a hydraulic servomotor employing a Moog type 71-101 valve as the control unit and a type 2H Hannifin cylinder as the linear actuator. The block diagram of the system is shown in Figure 1.

The building of a 3000 p.s.i. power supply was necessary and the design of this supply is considered in Appendices A and B.

The block diagram shows the basic units (excluding the power supply) that make up the system. They are the electrohydraulic servovalve, the linear actuator, a position transducer, and electronic amplifiers for producing a controlling error signal and supplying sufficient current to operate the servovalve.





### 2.3 The Electrohydraulic Valve

The valve used in this project was a Series 71-101 Moog Industrial servovalve. It was one of two such valves purchased with the intention to use them eventually in a numerically controlled drill press. Manufacturers specifications of the valve are found in Appendix C.

Typical load flow-load pressure curves for the valve are shown in Figure 2. as taken from the manufacturers specifications. The curve approximates the theoretical curve for an orifice where flow is proportional to the square root of the pressure drop across the valve.

The Moog 71-101 electrohydraulic valve is a two stage hydraulic amplifier in which the output stage is a four way closed centre spool valve and the first stage is a balanced flapper valve. The flapper of the first stage is positioned by an electric torque motor. A schematic of the valve is shown in Figure 4.

The torque motor consists of a permanent field magnet with permeable pole pieces. Between the pole pieces is an armature of soft iron around which are wrapped two solenoids, connected so that the fields oppose each other. When the current through each solenoid is the same, the solenoids contribute nothing to the magnetic flux through the armature, but when there is a differential current through the solenoids the flux through the armature is either increased or decreased. The change in flux produces



an imbalance in the forces between the armature and the pole pieces of the permanent magnet. This force imbalance produces a deflection of the armature. The armature of the torque motor is collinear with the flapper of the first hydraulic stage, and is held in position by a separation diaphragm of spring bronze, which exerts a returning force on the armature.

The operation of the flapper valve can be explained by referring to Figure 5. When the flapper is centered, flow through nozzles 1 and 2 are equal and flow through orifices 3 and 4 are also equal.  $P_1$  is then equal to  $P_2$ . If the flapper moves to the right a small amount, the flow through nozzle 1 increases and flow through nozzle 2 decreases. Flow through orifice 3 increases and thus the pressure drop across the orifice will increase, causing  $P_1$  to become smaller. In the same way  $P_2$  becomes larger. If  $P_1$  and  $P_2$  are applied to opposite ends of the spool of the second stage of the valve, the force developed by the differential pressure ( $P_1 - P_2$ ) on the ends of the spool will act against the restraining spool-centering springs and the spool will move. This movement of the second stage spool is then amplified in the following manner.

Figure 6. is a diagram of the second stage of the amplifier.

The springs at either end of the spool normally hold the spool in the centre position so that all orifices



are completely blocked by the lands of the spool. This is known as a closed centre valve. The only flow is the leakage flow past the lands. When a differential pressure appears across the spool the spool will move away from the central position. Assume that  $P_2 > P_1$ . The spool will then move to the left until the forces on the spool are balanced. Part of orifice 1 is uncovered and the same amount of orifice 4 is also uncovered. High pressure oil will flow into the right hand chamber of the valve and from there to the right port of the linear actuator. The piston of the actuator will be forced to the left by the flow through the right hand port. Oil on the left side of the actuator piston then flows out through the left port of the actuator and into the left chamber of the spool valve. From there it flows through orifice 4 and is returned at low pressure to the system reservoir. If the spool moves to the right, (i.e.  $P_1 > P_2$ ) orifices 2 and 3 are uncovered and the direction of flow is reversed. The orifices are designed so that flow through the orifice is directly proportional to the position of the spool.

A typical flow gain curve taken from manufacturer's specifications is shown in Figure 3. The curve represents the valve flow in terms of percent of rated flow as a function of percent change in rated current from the null current.







## 2.4 The Linear Actuator

The second part of the hydraulic motor is a linear actuator. The actuator used was a Hannifin Type 2H cylinder. It was chosen because of its pressure rating.

When there is no load on the actuator, the pressure on each side of the piston is about one half of the supply pressure, that is, 1500 p.s.i. assuming a supply pressure of 3000 p.s.i.. Thus, for some types of operation, a cylinder with a rated working pressure of about 2000 ps.i. would have been sufficient. However, when large loads are put on the actuator, the load pressure drop becomes greater and if a load large enough to stall the actuator is used, the full 3000 p.s.i. supply pressure is applied to one side of the piston. The piston must be capable of withstanding this pressure. Also, in some applications there are liable to be sudden reversals of flow, particularly in discontinuous operation. This will lead to shock pressures which may be several times normal operating pressure.

For these reasons the Hannifin Type 2H cylinder was chosen. It has a rated working pressure of 3000 p.s.i. and can be operated with shock pressures up to 5000 p.s.i.

In choosing the size of the cylinder there were two considerations.

The first was the flow capacity of the cylinder. The no-load flow of the Moog valve with a 3000 p.s.i. drop across the valve was 4 gallons (U.S.) per minute. It was



desired to obtain an appreciable stroke amplitude without surpassing the maximum flow that the Moog valve would permit. For this reason the actuator had to have a small volume displacement per unit length of stroke.

The second consideration was the need to produce the large force on the piston needed to give a fast system response. This required a large piston area.

A double ended piston rod was needed in order that the area of each side of the piston should be the same. This was necessary because of the symmetrical nature of the Moog valve.

A compromise was made between the need for large force and for low displacement. The cylinder chosen has a bore of 1 1/2 inch diameter and a rod diameter of 1 inch. The effective piston area is .982 square inches. The stalling force on the piston is 2940 pounds. When the actuator is operated at 1 c.p.s. with an 8 inch stroke its displacement is 4.08 gallons (U.S.) per minute. At 10 c.p.s. with a .5 inch stroke the displacement is 2.55 gallons (U.S.) per minute.

The cylinder has flanges at each end so that it can be bolted onto a horizontal surface and held rigidly for linear motion with one degree of freedom.

## 2.5 Design of a Suitable Position Transducer

Several types of linear position transducer are available for use with hydraulic systems. A popular design is the type in which a movable ferromagnetic rod, which acts as the core of a transformer, is attached to the cylinder of





the hydraulic system. In this type of transducer the output of the transducer varies linearly with the position of the ferromagnetic core. The maximum stroke of such a device at the present stage of development is about five inches, making it unusable for the system under consideration which has a stroke of ten inches.

Another type of transducer is the wirewound linear potentiometer. This device is inherently noisy and its resolution is limited. The output cannot be differentiated to obtain a velocity signal without producing undesirable voltage spikes.

There are many other commercial position transducers in which the sensing unit uses various types of resistive and inductive elements. There are also optical transducers using photosensing devices.

Because none of the commercial devices was available locally and because the experience of designing such a device was felt to be valuable, the transducer was developed rather than being purchased.

The design requirements for the transducer were:

1. It must be linear and usable over a ten inch stroke.
2. The output must be smooth and as free of noise as possible.
3. Its sensitivity must be such that it can detect a change in position of .001 inches. This is necessary if the device is to be used in numerical control applications.





4. It must be insensitive to environmental change.

The first attempt to satisfy these requirements yielded a device consisting of a straight piece of nichrome wire glued to a strip of plastic and a silver contactor which was bolted to the dolly of the hydraulic system. This was not satisfactory because:

i) There was not a continuous contact between the wire and the slider and as a result, the output of the device was noisy.

ii) Friction between the slider and wire introduced a dead zone which resulted in poor accuracy.

iii) The slider arm was not rigid. This effect tended to increase the error of the device because of the considerable play in the linear position of the slider.

iv) The low resistivity of the wire and its low power capacity made it inherently difficult to obtain high sensitivity.

## 2.6 Variable Capacitance Transducer

The final design of the position transducer used a variable capacitor as the sensing unit. A sketch of the transducer is shown in Figure 9a.

The minimum capacitance of the unit, which occurs when the movable armature is fully withdrawn to the end of its run is  $33 \mu\mu\text{f}$ . The maximum capacitance, when the armature is at the other end of its run is  $167 \mu\mu\text{f}$ . The capacitance of the position sensing element is directly proportional to



the position of the armature. The capacitance characteristic is shown in Figure 8.

The variable capacitor is located in one branch of a diode-capacitor bridge to which a 50 kilocycle square wave signal is applied. The bridge output, after being filtered to remove high frequency ripple, is directly proportional to the position of the armature of the sensing unit.

The transducer is mounted colinearly with the cylinder of the hydraulic motor and the armature of the transducer moves with the piston of the cylinder.

The circuitry of the transducer is shown in Figure 9b and its transfer characteristic is shown in Figure 8.

From Figure 8 the sensitivity of the position transducer was found to be 32.5 millivolts/inch.

This solution is not yet the best solution possible, but with further refinement this type of device will probably satisfy the requirements for a numerical control system.

The problems to overcome are:

1. Noise - The 50 kilocycle ripple on the output of the transducer must be further reduced so that the device can be used to sense small changes in position. Sixty cycle noise at 4 millivolts peak to peak amplitude must be reduced.

2. At present, the noise level is too high to permit differentiation of the position signal to obtain a velocity signal.





3. The sensitivity of the unit could be improved by increasing the change in capacity per unit change in position and by increasing the voltage of the 50 kilocycle input signal.

## 2.7 Amplifiers

The amplifiers used for producing the error signal and amplifying both the transducer output and the error signal were, with one exception, vacuum tube D.C. amplifiers. They were part of an analog computer which was borrowed from an undergraduate servomechanisms laboratory for the duration of the project. The analog computer scale factor was 100 volts = 1 computer unit and the amplifier gains were variable over a range of .01 to 100, by use of plug-in input and feedback resistors. The amplifiers used were chopper stabilized.

As mentioned previously, there was one exception. The amplifier supplying current to the solenoid of the Moog valve was a Nexus transistorized D.C. amplifier, type FSL6. This was used because the computer amplifiers were unable to supply the current necessary to drive the torque motor. They were capable of supplying up to 3 milliamperes while currents as high as 25 milliamperes were required to drive the torque motor. Since, in the Nexus amplifier, the scaling factor was 10 volts = 1 computer unit, it was necessary to scale down the voltage of the computer amplifier immediately preceding the Nexus. This was done by giving the amplifier a gain of .1.





The analog computer was also used for simulation of the relay when the system was operated as a relay servo as discussed later in this report. An amplifier and special non-linear circuits were used to make a saturating integrator, which is a close approximation to an ideal relay.

## 2.8 Power Supplies

Besides the several power supplies in the analog computer, three power supplies were used in the system.

1. Two Barnes Electronics Corporation Model 30-15 supplies. These are floating 15 volt supplies which were used to supply the +15 and -15 volts necessary for the Nexus amplifier. Their rated output is 975 milliamperes.

2. A Hewlett-Packard Model 721A transistorized supply. This was used to supply bias current to one solenoid of the torque motor. It is variable from 0 to 30 volts and has a maximum output of 225 milliamperes.

## 2.9 Actuator Load

In order to obtain dynamic characteristics of the system under operating conditions it was necessary to be able to put an inertia load on the system. To note the effect of system load, it was also desirable that the load could be varied.

Figure 7 shows the device that was used as a system load. It is a flat steel bed running on four ball bearings. The dolly can be bolted directly onto the piston rod of the actuator through a swivel.



It runs on a .5 inch thick steel plate, the level of which can be adjusted by means of six screws. This facilitates alignment of the components.

The load is composed of 4 lead ingots weighing 20 lbs. each. There is a fifth ingot which is lighter than the others and the weight of this ingot, plus the weight of the dolly totals 20 lbs. The ingots are held in place on the dolly by means of two long bolts threaded into the bed of the dolly. A piece of 1/4 inch strap bolted tightly against the top of the ingots prevents the load from shifting when the direction of motion is reversed.

With this device the inertia load of the system can be varied from that of the weight of the piston and rod alone to a load of 100 lbs. in 20 lb. steps.

The only damping in the system is that of the actuator. No additional external damping device was used.



## CHAPTER III

Analysis of Linear Operation3.1 General

Figure 1 is a block diagram of the hydraulic system. It is very helpful in the analysis of system operation if a mathematical equation expressing the theoretical relationship between the input and output of each block can be derived. The usual form of this relationship, which is called the transfer function of the block, is an algebraic equation employing the Laplacian operator. This expression is found by obtaining the differential equation of the device represented by the block and applying Laplace transformations to the equation assuming zero initial conditions. If this sort of expression can be obtained for each block in the system, then much can be discovered about the system stability, transient response, and steady state positional and tracking accuracy by employing the standard methods of linear analysis --- root locus, Nyquist diagrams, frequency response plots, and Nichols diagrams.

Of course, in the practical system linearity of components is sometimes a gross oversimplification. In this case, the transfer function obtained may apply for a specific set of operating conditions and for small signal operation. It is then necessary to obtain a number of transfer functions for a given block, each function applying to a different set of operating conditions.







In this chapter each block of the system is discussed and an approximate transfer function is derived for each. Various types of nonlinearity are discussed in order to show why experimental results may not agree with the theoretically derived linear approximations.

The analysis is divided into two parts, static characteristics and dynamic characteristics. The static analysis describes steady-state flows, pressures and forces in the system. The dynamic analysis describes the transient behaviour of the system.

### 3.2 Block Diagram

The block diagram of the closed loop system is shown in Figure 1. It is a typical position control system. An actuating signal,  $E(s)$  drives the current amplifier which in turn drives the torque motor. The armature of the torque motor positions the flapper of the first stage hydraulic amplifier which produces a differential pressure to drive the spool valve. The spool valve, in turn, drives the actuator. The output of the position transducer is a voltage proportional to the position of the actuator. This voltage is amplified and then subtracted from the reference signal. When the amplified transducer output is equal to the reference signal, the actuating signal will be equal to zero and the system will have reached a state of static equilibrium.



## STATIC CHARACTERISTICS

### 3.3 The Orifice Equation

The two variables in a hydraulic system are pressure and flow. If an analogy were to be drawn between an electrical system and a hydraulic system the voltage of the electrical system would correspond to the pressure of the hydraulic system and current would correspond to flow. The resistances of the electrical system correspond to orifices in the hydraulic system.

The preceding is only a very rough analogy. In an electrical system, components behave in a much more linear fashion than do the components of a hydraulic system. The analogy holds only in that a pressure differential will cause a fluid to flow in the direction of decreasing pressure. For the electrical system the equation  $V = iR$  describes the current flow through a resistor. This equation indicates that there is a linear relationship between current flow and voltage across a resistor.

For the orifice of the hydraulic system, the linear relationship does not hold. The equation describing flow through an orifice is:

$$Q = C_d A \sqrt{\frac{2 \Delta P}{\rho}} \quad (1)$$

$Q$  = volume flow

$C_d$  = orifice coefficient of discharge

$A$  = area of orifice



$\Delta P$  = pressure drop across orifice

$\rho$  = density of fluid

This equation is the standard orifice equation as derived from classical theory in any hydraulics text. It indicates that the pressure drop across an orifice is proportional to the square of the flow through the orifice.

The orifice constant, very roughly analogous to electrical resistance, depends on the area of the orifice, the density of the hydraulic fluid, and the orifice coefficient of discharge, which is in turn dependent in the geometry of the orifice and the Reynolds number. The Reynolds number is a nondimensional coefficient which is a function of the flow velocity through the orifice, the dimensions of the orifice, and the viscosity of the fluid. Since both the density and viscosity of the fluid are temperature sensitive, the orifice constant will be temperature sensitive.

The orifice discharge coefficient  $C_d$  is theoretically independent of pressure and temperature when the Reynolds number is high enough to ensure turbulent flow through the orifice. In order to make the Moog valve characteristics as reliable as possible the orifice dimensions are small enough to give a flow velocity that will guarantee turbulent flow through the orifice.

In a valve with rectangular ports such as the Moog Type 71-101, the port area is proportional to the deflection of the spool. If this deflection is indicated by the







variable  $x$ , the area of the orifice is then  $A = wx$  where  $w$  is the width of the orifice and is a constant.

The orifice equation then becomes:

$$Q = C_v \times \sqrt{\Delta P} \quad (2)$$

where  $C_v$  the orifice constant is found from:

$$C_v = C_d w \sqrt{\frac{2}{\rho}} \quad (3)$$

The above equations will be used later in this section to describe the operation of the Moog valve.

### 3.4 Amplifiers

The amplifiers used in the closed loop system of Figure 1 were high gain inverting D.C. amplifiers used as operational amplifiers. The transfer function for the amplifiers, if the gain is sufficiently high, can be found from the equation:

$$\frac{E_o}{E_i} = - \frac{Z_f}{Z_i} \quad (4)$$

where

$E_o$  = output voltage

$E_i$  = input voltage

$Z_f$  = feedback impedance from output to summing junction.

$Z_i$  = input impedance from input to summing junction

Note that there is a  $180^\circ$  phase shift from input to output as indicated by the minus sign.

The amplifiers used were wide bandwidth devices and since the feedback and input impedances were purely



resistive and the system as a whole was a low pass system the corner frequencies of the amplifiers can be considered too high to have any effect on the system and the transfer function can be represented by the equation:

$$\frac{E_o}{E_i} = -A \quad (5)$$

Where

$$A = \frac{R_f}{R_i}$$

### 3.5 Torque Motor

Figure 5 shows a simplified sketch of the torque motor. The analysis is based on the following operation.

When  $i_1$  is equal to  $i_2$  the solenoids do not contribute to the flux because they are wound in opposing directions. However, when  $i_1$  and  $i_2$  are not equal the net flux contribution of the solenoid is not zero. Flux in path I will be different from that in path II. The effect is to produce an unbalance in the forces on the armature, which produces a deflection of the armature. This, in turn, produces a deflection of the flapper in the balanced flapper valve.

The force on the armature due to the north pole of the magnet is

$$F_N = \frac{H_1 AM\mu}{d^2} \quad (6)$$

and the force on the armature due to the south pole is



$$F_S = \frac{H_2^2 A \mu}{(L-d)^2} \quad (7)$$

The field strengths between the armature and the pole pieces are

$$H_1 = \frac{N(\Delta i)}{R_L A \mu} \quad (8)$$

$$H_2 = \frac{N(-\Delta i)}{R_L A \mu} \quad (9)$$

$N$  = number of turns in each solenoid

The force on the armature is

$$F_A = F_N - F_S = \frac{N(\Delta i)M}{R_L d^2} + \frac{N(\Delta i)M}{R_L (L-d)^2} \quad (10)$$

where

$F_A$  = total force on armature

$F_N$  = force of attraction to north pole

$F_S$  = force of attraction to south pole

$H_1$  = field strength between armature and north pole

$H_2$  = field strength between armature and south pole

$A$  = area of pole pieces

$M$  = strength of permanent magnet

$\mu$  = permeability of air

$L$  = distance between pole pieces minus thickness  
of the armature

$d$  = distance from pole face to armature

$R_L$  = reluctance of the flux circuits

$\Delta i = i_1 - i_2$ , the differential current in the  
solenoids.





When the armature is centered,  $d = \frac{L}{2}$

and

$$F_A = \frac{8N (\Delta i) M}{L^2 R_L} \quad (11)$$

Let  $y$  be the displacement of the armature from the central position.

$$\text{Then } y = \frac{L}{2} - d \quad ; \quad d = \frac{L}{2} - y$$

Substituting yields

$$F_A = \frac{N (\Delta i) M}{R_L} \left[ \frac{1}{\left(\frac{L}{2} - y\right)^2} - \frac{1}{\left(\frac{L}{2} + y\right)^2} \right] \quad (12)$$

There are two other forces acting on the armature of the torque motor, one due to the spring action of the separating diaphragm and the second due to the pressure feedback of the flapper valve. The analysis of these two forces is complicated but approximate equations can be derived and the analysis shows that the overall characteristic of the torque motor is linear.

If it is assumed that the displacement of the armature is small with respect to its length and further, that the spring obeys Hooke's law, both of which are reasonable assumptions, then the returning force of the spring can be written as

$$F_s = K_s y$$

$$F_s = \text{spring force}$$

$$K_s = \text{spring constant}$$



$y$  = displacement of the armature from the central position, measured in the air gap between the pole pieces of the motor.

When the armature of the torque motor moves from the central position, the flapper of the first stage pilot valve also moves (see Figure 5.). This changes the sizes of the orifices on each side of the flapper. The flow through one orifice decreases and the flow through the other increases. Thus a pressure difference develops across the flapper, causing a force which acts in a direction to return the flapper towards the center position.

Applying the orifice equation

$$Q = C_D A \sqrt{\Delta P} \quad (13)$$

$$C_D = C_d \sqrt{\frac{2}{\rho}}$$

to the flapper valve the following equations (derived in section 3.6) are obtained.

$$P_1 = \frac{P_S}{\frac{\left(\frac{x}{L'}\right)^2}{4} + \frac{\left(\frac{x}{L'}\right)}{2} + \frac{1}{2}} \quad (24) \quad ; \quad P_2 = \frac{P_S}{\frac{\left(\frac{x}{L'}\right)^2}{4} - \frac{\left(\frac{x}{L'}\right)}{2} + \frac{1}{2}} \quad (25)$$

where

$x$  = displacement of the flapper from the center position

$A$  = area of nozzle orifice

$F_F$  = force on the flapper due to differential pressure

$2L'$  = distance between nozzles minus the width of the flapper.



$P_S$  = supply pressure

$$F_F = (P_1 - P_2)A \quad (14)$$

$$F_F = P_S A \left[ \frac{1}{\left(\frac{x}{2L'}\right)^2 + \left(\frac{x}{2L'}\right) + \frac{1}{2}} - \frac{1}{\left(\frac{x}{2L'}\right)^2 - \left(\frac{x}{2L'}\right) + \frac{1}{2}} \right] \quad (15)$$

Looking at the above equation, it is seen that it is of the same form as equation (12). If the parameters are properly chosen, the nonlinearity in  $F_A$  will be partially cancelled by the nonlinearity in  $F_F$  because the latter force is a negative feedback force.

The result is that the transfer characteristic of the torque motor is nearly linear over its operating range.

### 3.6 Balanced Flapper Valve

The controlled orifices of the flapper valve, shown in Figure 5, are the cylindrical openings between the flapper and the nozzles.

The areas of these openings and the flows through them are

$$A_1 = \left(\frac{L'}{2} + x\right) \frac{\pi d_N^2}{4} \quad (16) \quad Q_1 = \left(\frac{L'}{2} + x\right) \frac{\pi d_N^2}{4} C_D \sqrt{P_1} \quad (17)$$

$$A_2 = \left(\frac{L'}{2} - x\right) \frac{\pi d_N^2}{4} \quad (18) \quad Q_2 = \left(\frac{L'}{2} - x\right) \frac{\pi d_N^2}{4} C_D \sqrt{P_2} \quad (19)$$

$d_N$  = diameter of nozzle.

The spacing between the nozzles must be such that the controlled orifice flow never becomes greater than the maximum flow that can pass through the nozzle. That is,  $Q_1$  or  $Q_2$  must not be greater than  $\frac{\pi d_N^2}{4} C_D \sqrt{P_1}$  or  $\frac{\pi d_N^2}{4} C_D \sqrt{P_2}$  respectively. If this were to happen, the metering orifice would be the nozzle rather than the space between the nozzle







and the flapper.

The flows through the orifices  $d_o$  are found from the equations

$$Q_3 = \frac{\pi d_o^2}{4} C_D \sqrt{P_S - P_1} \quad (20)$$

$$Q_4 = \frac{\pi d_o^2}{4} C_D \sqrt{P_S - P_2} \quad (21)$$

Design principles\* recommend that  $L = \frac{d_N}{8} = \frac{d_o}{8}$  for best linearity.

The flapper valve drives the spring loaded spool of the second stage amplifier. Thus the output of the flapper is not integrated by the second stage. For this reason, the load flow from the first stage,  $Q_L$ , becomes zero after a short transient flow.

Under equilibrium conditions

$$\frac{\pi d_o^2}{4} C_D \sqrt{P_S - P_1} = \pi d_o (L' + x) C_D \sqrt{P_1} \quad (22)$$

and

$$\frac{\pi d_o^2}{4} C_D \sqrt{P_S - P_2} = \pi d_o (L' - x) C_D \sqrt{P_2} \quad (23)$$

Solving for  $P_1$  and  $P_2$

$$P_1 = \frac{P_S}{\frac{(\frac{x}{L'})}{4} + \frac{(\frac{x}{L'})}{2} + \frac{1}{2}} \quad (24); \quad P_2 = \frac{P_S}{\frac{(\frac{x}{L'})}{4} - \frac{(\frac{x}{L'})}{2} + \frac{1}{2}} \quad (25)$$

---

\* Morse, A.C., Electrohydraulic Servomechanisms, McGraw-Hill, 1963, p. 33.



The pressure across the second stage spool,

$$P_C = P_1 - P_2$$

$$P_C = - \frac{16 \left(\frac{x}{L}\right)^4 P_S}{\left(\frac{x}{L}\right)^4 + 4} \quad (26)$$

If  $x \ll L$ , then  $\left(\frac{x}{L}\right)^4 \ll 4$

and the expression for the control pressure becomes

$$P_C \approx - 4 \left(\frac{x}{L}\right)^4 P_S \quad (27)$$

which is a linear equation. It can be seen then, that for small deflections of the flapper, the spool valve operates with a linear characteristic.

### 3.7 The Spool Valve

There are several forces acting on the spool of the second stage.

1. Control force - this force is equal to the product of the differential pressure across the spool ends  $F_C = P_C A_S$  (28)

2. Spring force - this force acts to return the spool to the equilibrium position and is proportional to the deflection of the spool from the null position  $F_S = kx$  (29)

where  $x$  = the displacement of the spool from the null position.

3. Bernoulli Force - due to the uneven pressure distribution in the valve chambers during flow, an uneven force distribution develops. This



force has a resultant in the opposite direction to the motion of the spool. It can be expressed

$$\text{as } F_B = C_B Q \sqrt{P_V} \quad (30)$$

where  $P_V = P_S - P_L$ , pressure drop across the valve orifice.

$P_L$  = Load pressure drop

$C_B$  = a constant depending on the geometry of the valve.

For equilibrium

$$\Sigma F = 0 = F_C - F_S - F_B \quad (31)$$

$$F_C = F_S + F_B \quad (32)$$

$$P_C A_S = kx + C_B Q \sqrt{P_S - P_L} \quad (33)$$

Solve for  $x$

$$x = \frac{P_C A_S}{k} - \frac{C_B Q}{k} \sqrt{P_S - P_L} \quad (34)$$

If linear operation is assumed in the first stage and torque motor, then  $P_C$  can be expressed as a linear

function of the differential current through the torque

$$\text{motor. } F_C = K_C \Delta i A_S = C_C \Delta i \quad (35)$$

$$x = \frac{C_C \Delta i}{k} - \frac{C_B Q}{k} \sqrt{P_S - P_L} \quad (36)$$

The flow through the valve orifice is

$$Q = C_V x \sqrt{P_S - P_L} \quad (2)$$

Substituting (36) into (2)

$$Q = \frac{C_V C_C \Delta i}{k} \sqrt{P_S - P_L} - \frac{C_V C_B Q}{k} (P_S - P_L) \quad (37)$$





Solving for Q

$$Q = \frac{\frac{C_V C_C \Delta i}{k} \sqrt{P_S - P_L}}{1 + \frac{C_B C_V}{k} (P_S - P_L)} \quad (38)$$

where

$C_V$  = the orifice constant

$C_C$  = the first stage amplifier gain in pounds  
(force) per milliamperere.

$k$  = spring constant of the spool centering springs  
combined, in pounds (force) per inch.

Note: Q flows through two orifices and equation (38) describes the effect of the two metering orifices, which are assumed to be identical. The two orifices are the orifices from the high pressure to the cylinder port and the orifice from the return port of the cylinder to the low pressure return line to the reservoir. The actual situation in the valve is described as follows.

The pressure drop across the first orifice is  $P_S - (\frac{P_S}{2} + \frac{P_L}{2})$  and that across the second orifice is  $\frac{P_S}{2} - \frac{P_L}{2}$ .  $P_L$  is the load pressure drop across the actuator piston if the system is not loaded, that is, if  $P_L = 0$ , the valve flow becomes

$$Q = \frac{C_V C_C \Delta i \sqrt{P_S}}{k + C_B C_V P_S} \quad (39)$$

The effect of loading is to reduce the pressure drop across the valve orifices, thus reducing the flow available to drive the actuator. This form of nonlinearity is discussed in section 3.10.



### 3.8 Position Transducer

The position transducer consists of a diode and capacitor bridge as shown in Figure 9b. One of the capacitors is variable, the capacitance being directly proportional to the position of the movable arm (see Figure 9a).

$$C_t = K_p y + C_o$$

where

$C_o$  = minimum capacitance of the transducer

$C_t$  = capacitance of the position sensing device

$K_p$  = constant of proportionality

$y$  = position of the hydraulic actuator

A 50 kilocycle square wave signal is applied across the bridge by a Hewlett-Packard 203A function generator.

The diodes are assumed to have zero impedance when they are forward biased and infinite impedance when reverse biased. The operation of the circuit is analyzed by drawing equivalent circuits for the positive and negative half cycles of the square wave input, Figures 10a and 10b.

For the negative half cycle, the equations describing circuit operation are

$$e_{C_2} = E_i = i_2 R_2 - i_o' R_L \quad (40)$$



$$e_{C_1} = E_i - \frac{1}{C_1} \int_0^t i_1 dt = i_1 R_1 + i_{O'} R_L \quad (41)$$

$$i_1 = i_O - i_2 \quad (42)$$

If  $R_1 = R_2 = R$

$$i_{O'}(t) = \frac{E_i}{R + R_L} \left[ e^{\left( -\frac{R + R_L}{RC_1(R + 2R_L)} t \right)} - 1 \right] \quad (43)$$

Similarly for the positive half cycle

$$i_{O''}(t) = \frac{E_i}{R + R_L} \left[ 1 - e^{\left( -\frac{R + R_L}{RC_2(R + 2R_L)} t \right)} \right] \quad (44)$$

The average value of the output voltage is

$$\bar{E}_O = E_i R_L \frac{R + 2R_L}{(R + R_L)^2} Rf \left[ C_1 - C_2 - C_1 e^{-k_1} + C_2 e^{-k_2} \right] \quad (45)$$

where

$f$  = generator frequency

$$k_1 = \frac{R + R_L}{2RfC_1(R + 2R_L)} \quad (46)$$

$$k_2 = \frac{R + R_L}{2RfC_2(R + 2R_L)} \quad (47)$$

If  $k_1 \gg 1$  and  $k_2 \gg 1$

the exponential terms will become negligible with respect to  $C_1$  and  $C_2$  and the expression for the output voltage becomes

$$E_O = E_i f \frac{(R + 2R_L)}{(R + R_L)^2} R_L (C_1 - C_2) \quad (48)$$

Since  $C_1$  varies directly with the position of the actuator the output voltage of the transducer will also





be directly proportional to the actuator position.

With the following transducer parameters

$$R = 15 \text{ k}\Omega$$

$$f = 50 \text{ kilocycles}$$

$$C_{1\max} = 150 \text{ }\mu\mu\text{f.}$$

$$R_L = 12 \text{ k}\Omega$$

and equation (46)

$$k_1 = 3.1$$

This is high enough to yield a linear transducer.

The measured control characteristic of the transducer is shown in Figure 8.

## DYNAMIC CHARACTERISTICS

### 3.9 Transfer Function of the Torque Motor

The solenoids of the torque motor consist of a resistance and an inductance in series. Thus they should act as low pass filter with a simple pole. The transfer function of the torque motor will be

$$\frac{x}{\Delta i} = \frac{k_1}{1 + \tau_1 s} \quad (49)$$

where  $\tau_1 = \frac{L}{R}$  and  $s$  is the complex frequency.

In the Moog valve,  $R \cong 200 \text{ }\Omega$  and  $L \cong 1 \text{ henry}$  under pressurized operating conditions. The coil inductance varies greatly with operating conditions and cannot be measured accurately. With the given constants,  $\tau_1$  is calculated to be .005 seconds, giving a corner frequency of 32 c.p.s.



The dynamic analysis of the torque motor and flapper valve is quite complicated. The armature and flapper are cantilever mounted and held in place by a spring bronze separating diaphragm. The analysis would involve partial differential equations which would be nonlinear and the solution is beyond the scope of this report.

### 3.10 Transfer Function of the Spool Valve

In this analysis, friction and damping have been neglected because, in comparison to the positioning and spring forces, the damping force is negligible. This is because of the highly precise machining tolerances of the spool and spool channel. The spool valve is then a simple spring - mass system whose differential equation is

$$f(t) = M_S \frac{d^2x}{dt^2} + kx \quad (50)$$

$$F(s) = \Delta P(s) A_S = (M_S s^2 + k) X(s) \quad (51)$$

But  $F(s) = \frac{C_C \Delta I}{1 + \tau_1 s} \quad (52); \quad \frac{X(s)}{\Delta I(s)} = \frac{C_C}{1 + \tau_1 s} \cdot \frac{k}{\frac{M_S}{k} s^2 + 1} \quad (53)$

The frequency curves for the Moog valve are shown in Figure 11 at valve pressure drops of 3000 p.s.i. and 500 p.s.i. as taken from the manufacturers specifications. From the phase angle curves it can be seen that the phase lag increases toward 180°, indicating that the valve characteristic has two or more poles. This agrees with the qualitative result of the analysis. Not enough of the valve parameters



are known to be able to evaluate the exact transfer function. According to the manufacturers curves (Figure 11) the valve has a bandwidth of about 130 c.p.s. operating at no load and with a supply pressure of 3000 p.s.i. With a load pressure drop of 2500 p.s.i. and 3000 p.s.i. supply pressure, the bandwidth is 60 c.p.s.

### 3.11 Transfer Function of the Actuator

For the purpose of this analysis it is assumed that the hydraulic fluid is incompressible and that damping is due to viscous friction only. The effect of compressibility is considered in section 3.12.

The flow through the orifices of the spool valve is a function of the spool displacement and the load pressure. The equation  $Q = C_V \times \sqrt{P_S - P_L}$  (2) describes the idealized flow. In general

$$Q = f(x, P_L) \quad (54)$$

This can be expressed as

$$dQ = \frac{\partial Q}{\partial x} dx + \frac{\partial Q}{\partial P_L} dP_L \quad (55)$$

If the partial derivatives are evaluated for a particular set of operating conditions, the equation becomes

$$Q = \left( \frac{\partial Q}{\partial x} \right) \bigg|_{\substack{x=a \\ P_L=b \\ Q=c}} x + \left( \frac{\partial Q}{\partial P_L} \right) \bigg|_{\substack{x=a \\ P_L=b \\ Q=c}} P_L \quad (56)$$

In equation (56) it is assumed that the system is linear over the operating range.





For perturbations about this set of operating conditions

$$Q = C_x x - C_p P_L \quad (57)$$

where

$$\left. \begin{aligned} C_x &= \left( \frac{\partial Q}{\partial x} \right) \\ C_p &= \left( - \frac{\partial Q}{\partial P_L} \right) \end{aligned} \right\} \text{evaluated at } \begin{aligned} x &= a \\ P_L &= b \\ Q &= c \end{aligned}$$

Neglecting compressibility of the hydraulic fluid, the flow into the cylinder has two components.

$$Q = Q_o + Q_L \quad (58)$$

where

$Q_o$  = incompressible flow which causes motion of the piston

$Q_L$  = leakage flow

$Q_o = C_b Dy$  because the flow into the cylinder produces a constant velocity of the piston.

The leakage flow is proportional to the pressure drop across the piston,  $P_L$

$$Q_L = L P_L \quad (59)$$

This includes the leakage flow across the centre land of the spool valve as well as the flow past the piston.

$$A = C_x x - C_p P_L = C_b Dy + L P_L \quad (60)$$

$$C_x x = C_b Dy + (L + C_p) P_L \quad (61)$$

$D$  is the Heaviside operator  
 $y$  = displacement of the actuator  
 $L$  = coefficient of leakage



The force on the piston is

$$F = C P_L \quad (62)$$

where

$$C = n_F A \quad (63)$$

$n_F$  = force conversion efficiency of the cylinder

$A$  = effective area of the piston

If the load on the system consists of a damped mass as is the case in the system under consideration,

$$F = M D^2 Y + B Dy = C P_L \quad (64)$$

Substitute the value of  $P_L$  from (64) into (61) and solve for  $x$ .

$$x = \left[ \frac{C_b C + B(L+C_p)}{CC_x} \right] Dy + \left[ \frac{(L+C_p)M}{CC_x} \right] D^2 y \quad (65)$$

Laplace transformation of the above equation gives the following transfer for the actuator.

$$\frac{Y(s)}{X(s)} = \frac{CC_x}{s \left\{ (L+C_p)Ms + [C_b C + B(L+C_p)] \right\}} \quad (66)$$

or

$$\frac{Y(s)}{X(s)} = \frac{K_a}{s(\tau_2 s + 1)} \quad (67)$$

where

$$K_a = \frac{CC_x}{C_b C + B(L+C_p)}$$

and

$$\tau_2 = \frac{(L+C_p)M}{C_b C + B(L+C_p)} \quad (69)$$



As would be expected, the actuator is an integrator with an additional pole due to the damped mass loading.

### 3.12 Effect of Compressible Hydraulic Fluid

If the oil in the hydraulic system is considered to be compressible with a bulk modulus,  $N$ , the fluid in the cylinder can be considered as a spring. The hydraulic system with a pure inertia load will then act as a spring-mass system. The natural frequency of such a system is

$$f_n = \frac{1}{2\pi} \sqrt{\frac{2k}{M}} \quad (70)$$

where  $k$  is the spring constant in lbs. (force)/inch.

For the hydraulic system

$$\text{Deflection} = y = \frac{\gamma/2}{A} \quad (71)$$

$$\text{Force} = N A \quad (72)$$

where

$\gamma$  = enclosed volume

$A$  = area of the piston

$N$  = bulk modulus of the fluid.

Thus the hydraulic spring constant will be

$$k_H = N A \cdot \frac{A}{\frac{\gamma}{2}} = \frac{2N A^2}{\gamma} \quad (73)$$

The hydraulic natural frequency is

$$f_H = \frac{1}{2\pi} \sqrt{\frac{2N A^2}{\gamma} \cdot \frac{2}{M}} = \frac{A}{\pi} \sqrt{\frac{N}{M\gamma}} \quad (74)$$





This is effectively a resonant frequency and when the frequency response curves of the system are measured, a resonance peak should be found at  $f_H$ .

The bulk modulus of Univis J43, a hydraulic fluid similar to that used in the system under consideration is  $270 \times 10^3$  p.s.i.

For the system

$$\gamma \approx 10 \text{ in.}^2$$

$$A = .982 \text{ in.}^2$$

$$N = 270 \times 10^3$$

$$M = \frac{100}{32.2 \times 12} = .258 \quad \frac{\text{lb. sec.}^2}{\text{in}}$$

$$f_H = \frac{.982}{3.14} \sqrt{\frac{270 \times 10^3}{.258 \times 3.1}} = 191 \text{ c.p.s.}$$

This resonance is at too high a frequency to be detected on the frequency response curves.

### 3.13 Resonance of Structure

Another phenomenon which could affect the frequency response curve is resonance at the natural frequency of the mechanical structure of the mounting frame. It would be difficult to analyze the structure to find its natural frequency because of its irregular shape. However, it was observed that there was vibration from about 4 c.p.s. to about 8 c.p.s. and it could be assumed that this was because of a resonance with the structure.



To avoid this sort of resonance it is necessary to make the natural frequency of the structure quite high. This is done by making the structure rigid and massive. A large concrete slab with the hydraulic components solidly bolted to it would probably satisfy this criterion.

### 3.14 Summary

From the preceding analysis, an overall transfer function for the system can be written. It will be of the form

$$G = A \cdot K_I \cdot \frac{K_t}{(1 + \tau_1 s) \left( \frac{s^2}{\omega_V^2} + 1 \right)} \cdot \frac{K_a}{s(\tau_2 s + 1)} \cdot K_P \quad (75)$$

where

$A$  = gain of error amplifier

$K_I$  = current gain of Nexus amplifier

$K_t$  = first stage hydraulic gain

$K_a$  = actuator gain

$K_P$  = gain of position transducer

$\tau_1$  = time constant of torque motor solenoids

$\tau_2$  = time constant of actuator load

$\omega_V$  = natural frequency of spring loaded valve spool.

### 3.15 Nonlinear Effects

There are several sources of nonlinearity.

1. Valve saturation - when the spool displacement is so large that the control orifice is completely uncovered, flow becomes a function of load pressure drop



alone. The maximum flow for a given supply pressure is obtained when there is no load and the orifices are completely open. This flow is the saturation flow of the valve and it governs the maximum velocity of the actuator piston.

2. Loading effect - The equation for valve flow is

$$Q = C_V \times \sqrt{P_S - P_L} \quad (2)$$

where  $P_S - P_L$  is the pressure drop across the valve orifices. This can be rewritten

$$P_S - P_L = \frac{Q^2}{(C_V x)^2} \quad (76)$$

which gives a parabolic characteristic for the ideal valve. When the load is increased with  $x$  held constant  $P_L$  increases and  $Q$  decreases. The effect of increasing the load is to reduce the maximum flow that can be obtained from the valve for a given supply pressure. This in turn reduces the maximum velocity that the piston can attain.

3. Hysteresis in the torque motor flux paths - This produces a phase lag in the system which is proportional to frequency.

4. Backlash in the position transducer - This also produces a phase lag proportional to frequency.

5. Stiction in the spool valve and actuator introduces a dead zone which produces attenuation. It also tends to limit the accuracy of the system.





## CHAPTER IV

Discontinuous Operation4.1 Principle

Figure 12 shows the hydraulic system under discontinuous or relay operation. The block representing an ideal relay has a discontinuous characteristic. When the actuating signal,  $E$ , is positive, the output of the error amplifier is negative, and the relay is connected to the  $-V$  supply. When the error signal is negative, the output of the relay is  $+V$ . The value of  $V$  is such that, when it is applied to the input of the control valve, it produces a saturation flow into one of the actuator ports. Thus the maximum correction velocity is produced in the actuator piston. When the actuating signal becomes small its value oscillates rapidly about zero and a square wave of relatively high frequency is applied to the input of the valve. This oscillation is known as a limit cycle. It may not be present if the relay has a dead zone.

4.2 Advantages of Relay Operation

1. Since operation of the valve is not linear the valve does not require carefully machined parts. It is therefore likely to be much less expensive than a linear valve.

2. The valve is not as likely to malfunction if foreign materials are present in the hydraulic fluid because of the larger clearances between valve components. Effectively, all that is needed is a two position spool valve, since



the spool will not rest in an intermediate position. Any dead zone needed to eliminate a limit cycle is likely to be present in the valve, so a two position relay is desirable.

3. Faster response is obtained. The maximum correction velocity is always applied to the system. This results in the fastest rise time possible for the system.

#### 4.3 Disadvantages of Relay Operation

1. Sudden reversals of flow cause shock waves to develop in the system.

2. Analysis is more difficult, although there are several methods of treating relay systems.

3. In practice, relays have some hysteresis and dead zone. These introduce phase shift and reduce the stability of the system. They also introduce a large initial overshoot.

#### 4.4 Analysis of Operation

Two of the usual methods of nonlinear analysis are the phase plane and describing function methods.

The analysis assumes an ideal two position relay. In practice, the limiting integrator used to simulate the relay approximates this ideal. A transfer function for the hydraulic motor

$$G_m = \frac{K}{s(s+\frac{1}{\tau_1})(s+\frac{1}{\tau_2})(s^2+\omega_V^2)} \quad (77) \quad \text{is assumed.}$$

Assume an input to the relay of  $e_i = A_m \sin \omega t$ .



#### 4.5 Describing Function Method

The output of the relay will be

$$e_o = \begin{cases} +V & 0 < \omega t < \pi \\ -V & \pi < \omega t < 2\pi \end{cases} \quad (78)$$

$e_o$  can be expanded in a Fourier series

$$e_o = \frac{4V}{\pi} \left( \sin \omega t + \frac{1}{3} \sin 3\omega t + \frac{1}{5} \sin 5\omega t + \dots \right) \quad (79)$$

If the system is a low pass system with a narrow bandwidth it may be possible to neglect the higher harmonic terms present in the output of the relay on the grounds that they will be highly attenuated by the linear system which follows. If the system output is nearly sinusoidal when the reference input in an experimental situation is sinusoidal, then the describing function analysis can be considered valid.

Neglecting higher harmonic terms in equation (79)

$$e_o \approx \frac{4V}{\pi} \sin \omega t \quad (80)$$

The approximate transfer function of the relay is then

$$G_r \approx \frac{4V}{\pi A_m} \quad (81)$$

The approximate open loop transfer function of the relay operated system becomes

$$G = \frac{4VK}{\pi A_m s \left(s + \frac{1}{\tau_1}\right) \left(s + \frac{1}{\tau_2}\right) (s^2 + \omega_V^2)} \quad (82)$$







The gain constant is inversely proportional to the amplitude of the input signal.

#### 4.6 Phase-Plane Method

If the system acts as a pure integrator, that is, if the poles other than the one at the origin can be neglected then the system bandwidth becomes too wide for the describing function method to be useful, because the higher harmonic components of the output are still present in amplitudes that cannot be neglected.

The phase plane method can be used in this case. The phase plane is a plot of velocity against position with  $y$  as the abscissa and  $\frac{dy}{dt}$  as the ordinate.

$$\text{Assume that in Figure 12a, } G = \frac{K}{s} \quad (83)$$

Then

$$\frac{c}{E'} = \frac{K}{s} \quad (84)$$

$$\frac{dc}{dt} = Ke' \quad (85)$$

$$\text{But } e' = V(\text{sign } e) \quad (86)$$

$$e = r - c \quad (87)$$

Combining yields,

$$\frac{dc}{dt} = KV \text{ sign}(c-r) \quad (88)$$

This equation gives the output velocity as a function of position.

Figure 12b shows the phase plane for equation (88) with input step functions of  $+r$  and  $-r$ .



When the reference changes by a step of  $+r$ , the velocity has the value  $+KV$  until  $r = c$ , when the limit cycle ABCD is established.

#### 4.7 Simulation of a Two Position Relay

Figure 13 shows the circuit used to simulate the ideal two position relay used in this project. The first section is a saturating integrator. When

$-E < e_o < +E$  it acts as an integrator with a constant of  $\frac{1}{R_i C} \text{ sec}^{-1}$ . If  $e_i$  is given a positive value,  $e_o$  increases at a rate of  $-\frac{e_i}{R_i C}$  volts/sec. until it reaches  $-E$ . Then diode 2 begins to conduct and the circuit becomes a simple lag network with a gain of  $\frac{R_f}{R_i}$  and a time constant of  $R_f C$ . If  $\frac{e_i R_f}{R_i}$  is greater than  $E$ , the output of the device will continue to increase to  $\frac{e_i R_f}{R_i}$ . If  $e_i$  has a negative value, the output rises to  $+E$  volts where saturation occurs.

The bridge limiter that follows the saturating integrator is used to limit the output of the device to  $\pm V$  volts where  $|V| < |E|$ . This device eliminates the upward drift in the output that occurs when  $\frac{e_i R_f}{R_i}$  is greater than  $E$ . In the circuit used

$$R_i = 100K \quad C = 10 \text{ pf.}$$

The constant of integration is  $10^6 \text{ sec}^{-1}$ . With  $V = \pm .5$  volts and  $e_i \pm .001$  volts the rate of integration is  $\mp 1000$  volts/sec. and the rise time is one millisecond. This is fast enough to approximate an ideal relay.



## CHAPTER V

Experimental Tests and Results5.1 General

It was desired to obtain data on stability, steady state accuracy, and transient response of the system in both linear and relay operation. For this purpose open and closed loop frequency response curves were taken, and the transient response of the system with a square wave input was measured. The phase plane curve of the relay system was also measured. The hydraulic pressure in the system was maintained at 3000 p.s.i. as measured at the pump. All measurements were made while the oil temperature was in the range 120°F to 130°F.

5.2 Open Loop Measurements - Linear Operation

From the open loop frequency response curves it is possible to obtain evidence of the system type, error coefficients, open loop gain and an approximate open loop transfer function.

The hydraulic servomotor is a type one system or integrator. When it is in an open loop, any signal that appears at the input is integrated. If even a small D.C. signal is present, the output position will drift and eventually run to its maximum stroke. Thus it is difficult to obtain response curves with the open loop system.

To overcome this difficulty it was decided to measure the transfer function of the servomotor with the







loop closed. The advantage of this is that drift is eliminated. The output position can be held constant and a sinusoid superimposed on it.

Figure 14a shows the configuration used in obtaining frequency response curves.

In taking the open loop frequency response curves, the gain from 3 to 4 was measured for frequencies over the range 4 c.p.s. to 15 c.p.s., with some readings outside this range where possible. The output of the oscillator at 1 was varied so that the sinusoidal actuating signal remained constant.

Data was taken with the peak to peak voltage at 3 equal to 2 volts, 3 volts, and 4 volts, with loads of 20, 60, and 100 lbs.

The voltages at 3 and 4 were applied to inputs of the Tektronix Type 502 dual beam oscilloscope and were compared on the scope to obtain magnitude ratios.

Normally, phase plots can be obtained by comparing the signal at 4 with the variable phase reference voltage of the oscillator, at 5. The phase of voltage 5 is varied until the signal is in phase with voltage 4 and a direct reading of phase shift can be obtained from a dial on the oscillator. This reading is actually the phase shift between 1 and 5. However in the measurements described above there is a phase shift between 1 and 3 and the phase shift measured is that from 1 to 4 rather than that from 3 to 4.



Sets of open loop phase shift plots were obtained from the closed loop frequency response by using the Nichols diagram in reverse. Usually on the Nichols diagram open loop gain is plotted against open loop phase shift and closed loop response is obtained from the superimposed constant loop gain and closed loop phase-shift curves. In this case the closed loop response was plotted on the Nichols diagram and the open loop response obtained. This was the only way that the open loop phase shift of the system could be accurately obtained.

## 5.2 Closed Loop Measurements - Linear Operation

The configuration of Figure 4a was again used for the closed loop measurements. The gain and phase shifts from 1 to 4 were measured on the Tektronix scope, the phase shift measured by use of the reference phase of the oscillator as described in section 5.1.

Curves were taken with peak to peak input voltages of 1v, and 2v, and loads of 20, 60, and 100 lbs.

In order to observe the variation in the closed loop frequency characteristic with a change in open loop gain, magnitude ratio and phase shift curves were measured with additional gains of 5 and of 10 introduced into the system.

## 5.3 Transient Response - Linear Operation

A low frequency square wave was superimposed on the reference input of the closed loop system. The frequency chosen was .1 c.p.s. which has a period substantially longer





than the time constant of the closed loop system. Shock waves developed due to rapid switching of flow from one port of the actuator to the other, and the hammering due to these shockwaves was quite pronounced with peak to peak input voltages higher than one volt. To prevent damage to the Moog valve the square wave input voltage was not made greater than one volt peak to peak.

Transient response curves were made using a Sanborn 150 recorder. Curves were taken for system loads of 20, 60, and 100 pounds with peak to peak inputs of 1 volt. Two other curves were made for the closed loop system with additional gains of 5 and of 10. The load was 20 pounds and the input was a .5 c.p.s. square wave with a peak to peak amplitude of .5 volts.

#### 5.4 Measurements on Relay System

Figure 14b shows the circuit used for testing the relay-controlled system.

For transient response measurements a square wave was superimposed on the reference at 1. The signal at 3, the input to the Moog valve, is proportional to velocity when the valve is operating in the linear region.

Because of the large shock waves encountered in switching high flows it was necessary to operate the valve in the linear region. In relay control this is not normally the case. The input current is usually that which produces the saturation flow and the maximum velocity is that velocity which corresponds to the saturation flow.





Operation in the linear region gave one unanticipated advantage. Since the input signal to the valve was proportional to the velocity of the actuator, it was possible to observe the phase-plane curve of the system with a step input.

Because of the relatively wide bandwidth of the servomotor harmonics were present in the output of the system in amplitudes that could not be neglected. For instance, output waveform of the system with a sinusoidal input was a near-perfect triangular wave. For this reason, the describing function method of analysis was not useful.

Two transient response curves were obtained for the relay system. The input in each case was a .1 c.p.s. square wave with an amplitude of one volt peak to peak. The load on the actuator was 20 lbs. For the first curve the relay output was  $\pm .5$  volts. For the second it was  $\pm .8$  volts. These curves were made for comparison with the transient response of the linear system under similar operating conditions

A complication which has interesting implications arose in the operation of the system as a relay servo. The output of the position transducer has a 60 cycle noise component of about four millivolts. This 60 cycle component was amplified by the error sensing amplifier and appeared at the input to the relay with an amplitude large enough to cause the relay to operate whenever the actuating signal



dropped to zero. The result was a 60 cycle square wave with a peak to peak amplitude of one volt applied to the input of the Moog valve.

It was also noted that when an error signal smaller in amplitude than the peak voltage of the 60 cycle signal was applied to the relay, the output of the relay was a width-modulated 60 cycle rectangular wave in which the difference in width between the positive and negative side of the wave was proportional to the error signal.

The 60 cycle square wave did not cause any measurable motion of the actuator although considerable vibration was felt in the structure.

The implication of this observation is in the discovery of a dual mode type of relay operation. Further development would involve eliminating the 60 cycle noise from the transducer and replacing it by a signal of a higher frequency, about 400 c.p.s., and low amplitude.

This signal would apply a dither to the valve to increase its resolution. It would also give two modes of operation. For small changes in the reference signal the Moog valve would be operated by a width-modulated 400 c.p.s. square wave. For reference changes larger than the amplitude of the 400 c.p.s. dither signal the Moog valve would be operated as an ordinary relay servo. This type of operation combines high resolution with rapid response.





## 5.6 Open Loop Results - Linear Operation

Figure 16 is a frequency response curve of the open loop system with a 4 volt peak to peak input and 100 lb. load. Only the magnitude ratio is plotted. The phase shift of the open loop system can be seen on the Nichols diagram, Figure 18, derived from the closed loop frequency response curve of Figure 17. Curves obtained for other operating conditions are identical to Figures 16 and 18 and are not included with this report.

The open loop frequency response curves indicate that the system is almost a pure integrator. The effect of the system load and the servovalve bandwidth is not evident in the range of frequencies in which measurements were made.

The open loop curves also show that little, if any, non-linear effects are present. The gain constant, the value of system gain at 1 rad/sec, does not vary with load or with the magnitude of the input signal. The phase shift remains constant at about 90°, as would be expected in a type one system at low frequencies. At the higher frequencies, above 1 cps, the phase lag seems to increase but no reliable phase shift data could be obtained above 3 cps because of noise.

The open loop transfer function of the servovalve can be given approximately by the equation

$$G(s) = \frac{1.25}{s}$$

The additional pole, due to the effect of the load, as predicted in section 3.11 did not appear. The pole was calculated to be at





$$\omega_c = \frac{C_b C + B(L+C_p)}{(L+C_p)M} \quad (90)$$

$C$  is the effective area of the cylinder which is approximately one square inch.

$$C_b = \frac{Q}{Dy}$$

The integration constant = 1 in<sup>2</sup>

$C_p$  is the slope of the load flow - load pressure curve at the operating point which is approximately  $10^{-3}$  in<sup>5</sup>/lb.sec.

$L$  is the leakage coefficient. According to manufacturers specifications, this is less than 3% of rated valve flow. If the leakage is neglected the corner frequency becomes

$$\omega_c \approx \frac{C_b C + BC_p}{C_p M} \quad (91)$$

If, in addition, the product  $BC_p$  can be neglected and this is likely the case, since system damping is low,

$$\omega_c \approx \frac{C_b C}{C_p M} \quad (92)$$

The maximum load encountered is  $.259 \frac{\text{lb sec}^2}{\text{inch}}$

The lowest corner frequency to be expected is then

$$\omega_c = 3850 \text{ rad/sec} = 610 \text{ c.p.s.}$$

From these rough calculations it can be seen that the system load must be much higher or the damping coefficient larger before the frequency response of the system will begin to deteriorate.



One additional constant can be obtained from the open loop curve. This is the velocity error coefficient, which is the point where the open loop response curve crosses the zero db. axis. It is found to be  $1.3 \text{ sec}^{-1}$ .

For a type one system, the position error coefficient is theoretically infinite, indicating that no constant actuating signal is needed to maintain the actuator in a fixed position. If there is an error in the actuator position it will be due to dead zone or hysteresis in the valve or actuator.

The sensitivity of the transducer is 32.5 millivolts/inch. The output of the transducer has a 60 cps. noise component with an amplitude of about 4 millivolts peak to peak.

If it is assumed that a signal with the same amplitude as the noise can be distinguished from the noise, a step change of 4 millivolts should be detectable. This corresponds to a change in position of .12 inches. With a D.C. voltmeter it is possible to detect smaller changes that would not be noticable on the oscilloscope. However, the noise level does make it difficult to determine precisely whether there is a steady state positional error.

Another factor contributing to the difficulty in measuring positional error is the error in amplifier gains due to the use in the input and feedback paths of resistors with a tolerance of  $\pm 1\%$



### 5.7 Closed Loop Results - Linear Operation

If the open loop transfer function of the servomotor is  $G = \frac{1.25}{s}$  the closed loop transfer function, according to linear theory is:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)} = \frac{1}{\frac{s}{1.25} + 1} \quad (93)$$

This indicates a corner frequency for the closed loop frequency response curve at .2 cps.

The closed loop frequency response curve for 2 volt peak to peak input and 100 lb. load is shown in Figure 17. The corner frequency is at .2 cps. There is some evidence of a second corner frequency beyond the highest frequency at which measurements were made but nothing conclusive. The time constant of the closed loop system is .8 seconds.

When the open loop gain is increased by a factor of 5, the bandwidth should increase to 1.0 cps. and it should increase to 2.0 cps. when the gain is increased by a factor of 10. Figures 19 and 20 show that this is indeed the case. The time constants for the system with additional gains of 5 and 10 are .16 sec. and .08 sec. respectively.

### 5.8 Transient Results - Linear Operation

Figures 21, 22, and 23 are tracings of the system output with a square wave input. Figure 21 was made with a .1 cps input of 1 volt peak to peak amplitude and 100 lbs. load. Figures 22 and 23 were made with a load of 20 lb. and







a .5 cps. input with amplitude of .5 volts peak to peak. For Figure 22 the open loop gain was increased by a factor of 5 and for Figure 23 it was increased by a factor of 10.

The time constants of the output waveforms of Figure 21 is .8 seconds. The time constants of the output waveforms in Figures 22 and 23 are .16 and .08 seconds respectively.

All these values are in agreement with the time constants calculated in section 5.7 from the open loop frequency response curves.

The transient response of the closed loop system could be further improved by increasing the open loop gain. An additional gain of 10 could have been added, increasing the closed loop bandwidth to 20 cps. and the time constant to .05 seconds.

The gain was not increased because of the large shockwaves that occurred during rapid switching of large flows.

The upper limit on the response speed is set by the maximum flow that can be supplied by the Moog valve. With rated differential current (15 milliamperes) and a 3000 p.s.i. valve pressure drop, the Moog valve can supply 4 gallons (U.S.)/minute. The saturation flow is larger than this but no instruments were available to determine its value.



## 5.9 Relay Operation - Results

Figures 24 and 25 are transient response curves for the relay operated system. The input to the system was a 1 cps. square wave with an amplitude of 1 v. peak to peak. Figure 24 represents the response when the relay output was  $\pm .5$  volts. Figure 25 is the response for a relay output of  $\pm .8$  volts.

The rise time, defined as the time from the application of the step to the point where the output first reaches its steady state value, is 1.9 seconds in Figure 24 and 1.1 seconds in Figure 25.

The rise time was used here as a performance criterion rather than the time constant, as used in the linear case, because the output of the relay does not respond exponentially. The output responds to a step input with a rapid rise at a constant velocity and an almost negligible overshoot.

On comparing the speeds of response of the linear and the relay-controlled systems the difference between rise time and time constant should be noted. Since the step response of the linear system is overdamped, the output does not reach its steady state value until several time constants after the step input is applied. Thus a time constant of .8 seconds is equivalent to a rise time of more than 1.5 seconds. Taking this into consideration it is then evident that the transient response of the relay-controlled system is as fast as that of the linear system.



The phase-plane curve of system response to a step input was observed on a Tektronix Type 502 Dual Beam oscilloscope and found to be identical to Figure 12b.

The valve was not operated in saturation because of the shock waves which developed during switching. A relay output voltage greater than  $\pm 3.0$  volts is necessary to operate the valve in saturation. Operation in the saturation region would result in an improvement in rise time of three to four times, that is, a rise time of about .2 seconds. The shock waves could be eliminated or greatly reduced by use of a surge damping valve in the lines between the valve and the actuator.







## CHAPTER VI

Conclusions

The immediate objective of this project, the building and testing of a hydraulic motor to operate at 3000 p.s.i. has been attained. There is still much work to be done and much information to be obtained before the development of a machine tool control system can be attempted. Some of this is discussed in Chapter 7.

The conclusions reached are:

1. The system is linear in the range of operating conditions over which it was tested. No loading or saturation effects were noted.

2. The open loop system acts as a pure integrator. There is an indication that there is a second corner frequency at some frequency above the range over which testing took place but it is high enough for its effect to be neglected. The closed loop system is therefore inherently overdamped.

3. The relay controlled system can be made to respond to a step input as quickly as the linear system. The reason is that the speed of response in both cases is limited by the maximum valve flow. In effect, the linear system is not actually linear, and as the system gain is raised, the actuating signal eventually becomes large enough to raise the valve flow to its saturation level.



Recommendations

1. The base of the servomotor should be cast in one piece and the surface ground. This would facilitate alignment of components.
2. The actuator and the position transducer should be made in one unit. Most commercial actuators are made in this way. It would help eliminate backlash in the transducer and make electrical shielding easier.
3. The base of the servomotor should be fastened rigidly to a heavy concrete slab with a high natural frequency to eliminate mechanical resonance.
4. The flow capacity of the oil cooler should be increased. When there is a high flow demand the pump appears to be starved. Cavitation in the pump produces shock waves and hammering.
5. The packings of the Hannifin cylinder may not be compatible with the hydraulic fluid. Nor are they capable of standing prolonged operation at frequencies above 1 cps. The cylinder should be modified to take heavy duty packings and compatibility of the hydraulic fluid should be investigated.
6. The gain and sensitivity of the position transducer should be improved.
7. There is 60 cycle noise of about 4 mv. peak to peak amplitude superimposed on the output of the transducer. This makes it difficult to extend the frequency response measurements beyond 15 cps. The 60 cycle signal also causes



difficulties in relay operation (see section 5.4). This noise should be eliminated or greatly reduced.

8. A surge damping valve should be inserted between the actuator and the Moog valve to reduce the hammering during rapid switching of flow.

9. The actuator should be tested with loads much higher than 100 pounds and with larger damping. In the present arrangement 100 pounds is the maximum load that can be applied to the system.

10. A flow meter should be obtained so that the flow characteristics of the Moog valve can be accurately determined over a wide range of operating conditions.





## BIBLIOGRAPHY

1. Fluid Power Directory, 1962-63 Edition.
2. Blackburn, J.F.; Reethof, G; and Shearer, J.L., Fluid Power Control, The Technology Press of M.I.T., John Wiley and Sons, 1960.
3. Morse, A.C., Electrohydraulic Servomechanisms, McGraw-Hill, 1963.
4. D'Azzo, J.J.; Houpis, C.H., Feedback Control System Analysis and Synthesis, McGraw Hill, 1960.
5. Thaler, G.J.; Pastel, M.P., Analysis of Design of Nonlinear Feedback Control Systems, McGraw-Hill, 1962.



## APPENDIX A

High Pressure Power SupplyA.1 Requirements

## 1. Pressure -

a) The system was designed to operate with a working pressure of 3000 p.s.i.

b) It was desirable that the pressure be continuously variable from zero to the maximum pressure.

## 2. Flow -

a) At 3000 p.s.i. valve drop the rated flow of the Moog type 71-101 servovalve is 4 gallons (U.S.) per minute. The power supply had to be able to give enough flow for two of these valves. A value of 9 gallons (U.S.) per minute was decided upon in order to ensure adequate flow.

b) Lines and control valves should not have flow restrictions when open.

## 3. Variable displacement -

a) The system does not demand full flow capacity at all times. Any excess flow is returned directly to the reservoir through the relief valve. This constitutes unnecessary waste of power and greatly reduces the efficiency of the system.

e.g. If the pump has a fixed displacement of 9 gal. (U.S.) per minute and the system demands only 3 gallons (U.S.) per minute, 6 gallons (U.S.) per minute returns to the reservoir through the relief valve. With





a pressure drop of 3000 p.s.i. across the relief valve there is a loss of 10.5 h.p. in the relief valve.

b) Excessive flow at high pressure causes heating of the oil. In the preceding example, power developed in the relief valve by the return flow produces 7.42 B.T.U. per second.

The pump purchased has an automatic pressure-controlled variable displacement compensator.

4. Pressure fluctuation in the system was undesirable. A 5 gallon accumulator was available and was teed off the main supply line to help maintain a constant pressure by supplying large transient flow demands and damping out shock waves.

#### 5. Shock Waves -

a) Fast switching of the servovalve generates shock waves. These must be prevented from reaching the pump. The surge damping valve in the main supply line does this.

b) The accumulator acts as a cushion against shock waves.

#### 6. Safety -

a) Overpressure protection was considered essential in the system. A pressure switch was used for this purpose.

b) All power supply components were designed for 3000 p.s.i. working pressure, with a safety factor of at least 3.



c) Operator error in use of controls should not cause accidents.

d) Pressure and temperature gauges must be clearly visible.

#### 7. Temperature -

a) Fluid temperature must be held below 160°F to prevent excessive vapourization and fire hazard, and to prevent decomposition of the fluid, which causes formation of sludge.

b) The temperature of the system should be held as constant as possible because of the temperature sensitivity of the valve characteristics.

#### 8. Filtration -

a) The Moog valve specifications require that the system hydraulic fluid be filtered to remove particles greater than 10 microns in diameter.

b) There are special sintered bronze filters in the Moog valve to protect the flapper valve.

b) If the oil is to be filtered in the high pressure line a high strength filter is needed that will not shatter on impact. Because of the expense involved it is desirable to have the filter in the suction line between the reservoir and the input of the pump.

#### 9. Gauges -

a) It is necessary to be able to monitor the pressure at the pump (for setting the relief valve).

b) The temperature of the oil in the reservoir



should be monitored.

10. Control -

a) An on-off switch was needed to control the electric motor driving the pump. It was desirable that this should be a remote control station.

b) A solenoid valve was used to separate the pump from the system so that the pump could be run without pressurizing the system.

c) It was necessary to be able to discharge the accumulator rapidly into the reservoir. A special set of manually operated valves for diverting the main flow from the pump and accumulator to the reservoir, bypassing the servovalve, was built.

d) Controls should be convenient to operate and not liable to cause damage to the system through operator error.

11. If the pump is shut off while the accumulator is under pressure the oil stored in the accumulator will flow back through the pump, forcing it to run in reverse. This is undesirable because under this type of operation the pump sucks air into the system. A check valve at the outlet of the pump prevents flow return through the pump.

12. Leakage in the system should be held to a minimum.





- a) To prevent loss of fluid
- b) To prevent damage to the floors
- c) To eliminate accident hazard due to persons slipping on spilled oil.

#### 13. Reservoir -

A reservoir with a capacity of 20 Imp. Gallons was available. The suction line intake of the reservoir is raised six inches above the bottom of the reservoir to prevent intake of deposited sediment. The reservoir remains at atmospheric pressure but there are no openings through which contaminants from the atmosphere can come.

#### 14. Hydraulic Fluid -

The fluid used complies with the specifications for MIL-O- 5606.



## APPENDIX B

B.1 High Pressure Power Supply - Description of Components

The block diagram of the high pressure power supply is shown in Figure 15. Below is a description of the components.

1. Pump - A Denison Industrial Variable Volume Series 500 pump was used in the system. It is of axial piston, variable stroke design. It delivers a maximum flow of 9.7 gallons (U.S.) per minute at 1800 r.p.m. and zero p.s.i. At 3000 p.s.i. and 1800 r.p.m. it delivers 9.3 gallons (U.S.) per minute. It is rated at 3000 p.s.i. maximum working pressure and 4500 p.s.i. intermittent pressure.

The compensator of the pump adjusts the flow automatically by means of pressure feedback. The angle of the pump's cam plate is varied by this feedback so that the pump will idle when the flow demand is not great.

2. Electric Motor - The pump is driven by a Westinghouse Lifeline A motor. The motor runs on a 60 cycle, 3 phase, 208 volts/phase supply and can deliver 20 horsepower at 1760 r.p.m.

3. Relief Valve - The relief valve is a Denison Series R2 pilot operated valve. The valve is Teed off the main supply line and the low pressure oil passing out of the valve is returned to the reservoir through a one inch hose line. With the relief valve, the pressure at



the pump can be maintained at any value between 0 and 3000 p.s.i. Since the valve is not in the main line it does not introduce a flow restriction.

4. Check valve - An Enoco MT50 check valve was placed in the main supply line to prevent oil from flowing back through the pump when the pump was shut off. In order to reduce the pressure drop across the valve for flow in the forward direction (i.e. toward the servomotor) a 1/2 inch valve was chosen.

5. Pressure Switch - To protect the system from pressures higher than the rated working pressure a pressure switch was used. The switch is a Barksdale B1TA32SS normally closed Bourdon tube switch. It operates off the main supply line on a 1/4 inch stainless steel pilot line and will trigger at preset pressures from 188 to 3200 p.s.i. When the switch is triggered, it opens and the electric motor shuts off.

6. Gauges -

a) Pressure gauge - A Marsh Type 100 gauge was used to monitor pressure at the pump. It operates off the main supply line on a 1/4 inch hose pilot line. The gauge is of Bourdon tube design with steel movement to prevent excessive wear. Sudden variations in pressure are damped out with a Type DS diaphragm attachment. The gauge has a 4 1/2 inch diameter dial face and is calibrated from 0 to 5000 p.s.i.

b) Temperature gauge - A thermocouple thermometer monitors the oil temperature in the reservoir. The temperature







is displayed on a Weksler gauge calibrated from 30° to 240°F. in 2 degree intervals. The gauge has a circular 4 inch diameter dial face.

7. Two Way Solenoid Valve - The pump is isolated from the rest of the system by an on-off solenoid operated valve. The valve, a Fluid Power Accessories 3-C5, is normally closed and is activated by a 110-volt, 60 cycle signal. It operates in the main supply line and has a 3/4 inch orifice. When open, it offers little resistance to flow.

8. Surge Damping Valve - In order to prevent surges and shock waves from being transmitted from the servomotor back to the pump a Denison Hydraulic Surge Damping Valve was inserted in the main supply line. The valve automatically adjusts itself to any working pressure up to 5000 p.s.i. but closes rapidly to prevent passage of shock waves in the reverse direction to normal flow.

9. Accumulator - To help smooth the pressure applied to the control valve of the servomotor and to supply occasionally required flow surges an accumulator was teed off the main line. The accumulator used was a Greer A105-200 5 Gallon (U.S.) bladder type hydraulic accumulator. The cushion in the bladder was nitrogen at 1000 p.s.i. The pilot line was 3/8 inch stainless steel tubing.

10. Filter - Elimination from the hydraulic fluid of particles down to 10 microns in diameter was



accomplished by inserting a Greer Low Pressure Micronic Filter in the suction line of the pump. The filter is rated at 500 p.s.i. maximum pressure and 30 gallons (U.S.) per minute. Additional filtration takes place in two sintered bronze filters found in the body of the Moog valve, and a larger pilot stage filter also in the valve.

11. Oil Cooler - It was possible to hold the temperature of the hydraulic fluid below 130°F. for all types of operation encountered in this project by means of an oil cooler constructed in the Electrical Engineering Machine Shop. The oil flows through a 4 inch diameter copper pipe inserted in the suction line. The cooling is done by tap water flowing through to parallel connected spiral coils of 3/8 inch tubing located within the large copper pipe. Each coil was wound from 50 feet of soft copper tubing. The oil flow is opposite to the water flow and the oil is forced to flow around the copper tubing by baffles welded to a 2 inch diameter copper pipe, sealed at either end, which runs up the centre of the larger cylinder. There is a possibility that for higher flow than that required for this project the restriction introduced into the suction line by the oil cooler may be too great and the cooler may have to be modified.

12. Other valves -

a) Accumulator discharge - A Jamesbury 3/8 inch Type HP22GT manual shutoff valve was inserted off the main





supply line on a 1/2 inch hose returning directly to the reservoir. The purpose of this valve was to enable the operator to discharge the accumulator rapidly into the reservoir. The Jamesbury valve is a ball valve with a 90° movement of the stem from the fully open position to the fully closed position. The maximum working pressure of the valve is 4500 p.s.i. It has nylon seats and teflon seals.

b) Flow Metering - A Marsh 1/2 inch 1900MFA needle valve with a maximum working pressure of 10,000 p.s.i. was inserted in the main supply line just before the Moog valve. With this valve the accumulator can be isolated from the servomotor and the flow to the servomotor can be accurately metered. The valve, when fully opened, does not introduce much restriction into the line.

13. Fittings and Transmission Lines- Where components were expected to remain permanently fixed with respect to each other, stainless steel tubing was used for transmission of hydraulic fluid. Imperial self-sealing tube fittings were used with all the stainless steel tubing. These fittings have ferrules which are compressed onto the tubing when the fitting is tightened. The advantage of this type of fitting is that the tubing does not have to be flared at the ends. The disadvantages of the particular fittings used were that the tubing had to be aligned perfectly with the fitting to assure a leakproof seal and that the seals tended to leak if the connections were opened and then resealed as was required sometimes to vent the system.





In general, for short transmission lines not subject to excessive wear or abrasion, hose lines are the most flexible solution to transmission problems. They permit some rearrangement of components such as may be occasionally necessary for more convenient operation and they eliminate the work involved in bending and properly fitting stainless steel lines.

Most of the lines used in the hydraulic power supply and the servomotor are high strength, braided steel strengthened, hydraulic hose with straight thread 37° flared connectors.



## APPENDIX C

Moog Valve Specifications

Operating Supply Pressures, 500 - 3000 p.s.i.

Rated Input Signal (differential current), 15 ma.

Coil Resistance, (2 coils) each  $200 \pm 25$  ohms.

Rated flow with 1000 p.s.i. valve drop, 2.5 Gal. (U.S.)/min.

Temperature Range, 0 - 250°F.

Recommended System Filtration, 10 microns.

Resolution - The maximum increment of input current required to produce a change in valve output flow. - Less than 1% of rated current.

Hysteresis - The total difference in differential currents required to produce zero load flow as the valve is cycled between rated positive and negative current.

It is measured with zero load pressure.

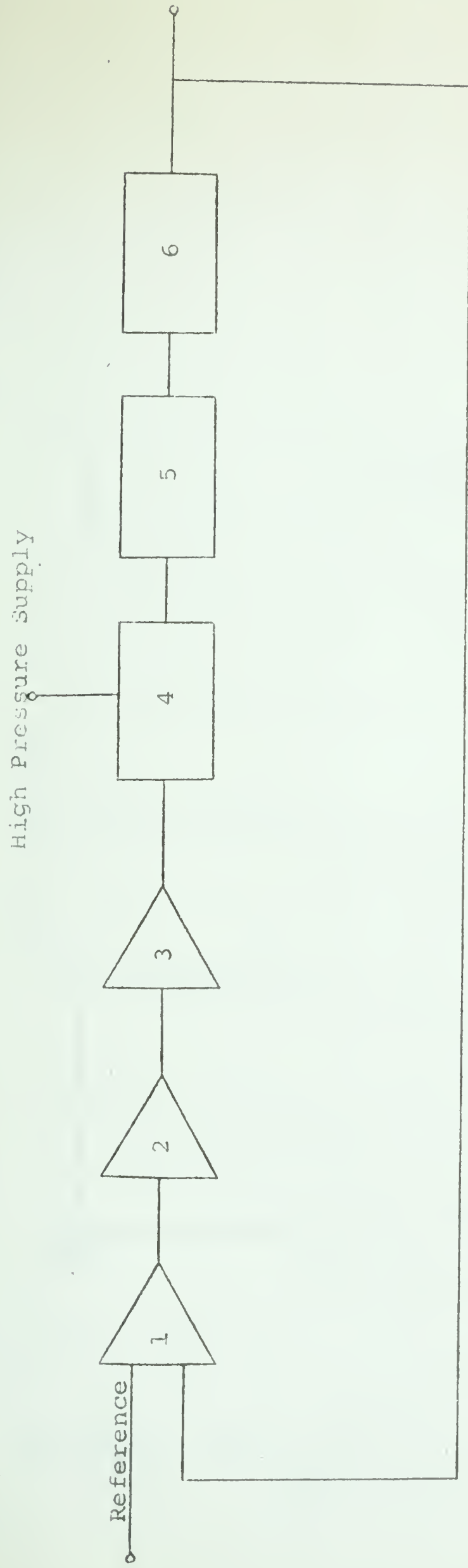
- Normally less than 4% of rated current.

Null Leakage - Composed of first stage flow and second stage null leakage.

- First stage flow is less than  $.35 \text{ in}^3/\text{sec}$ .

- Second stage flow is normally less than 3% of maximum flow at rated system pressure.





1. Summing Amplifier
2. Scaling Amplifier
3. Nexus Transistor Amplifier
4. Moog 71-101 Electrohydraulic Servovalve
5. Hannifin 2H Cylinder
6. Position Transducer

Figure 1  
Block Diagram of the Electrohydraulic System





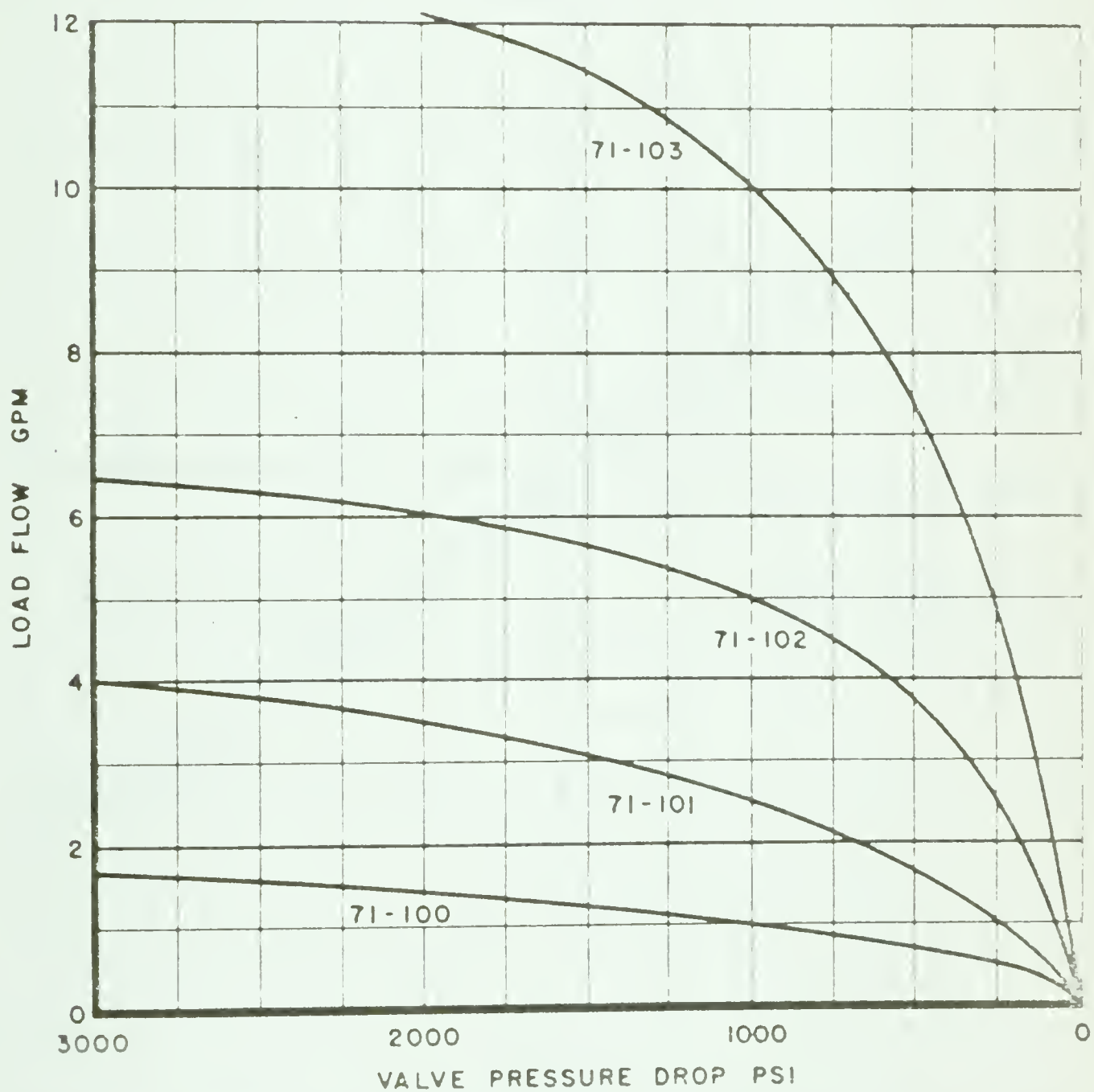


Figure 2  
Load Flow - Load Pressure Curve for Moog 71- 101  
Electrohydraulic Valve



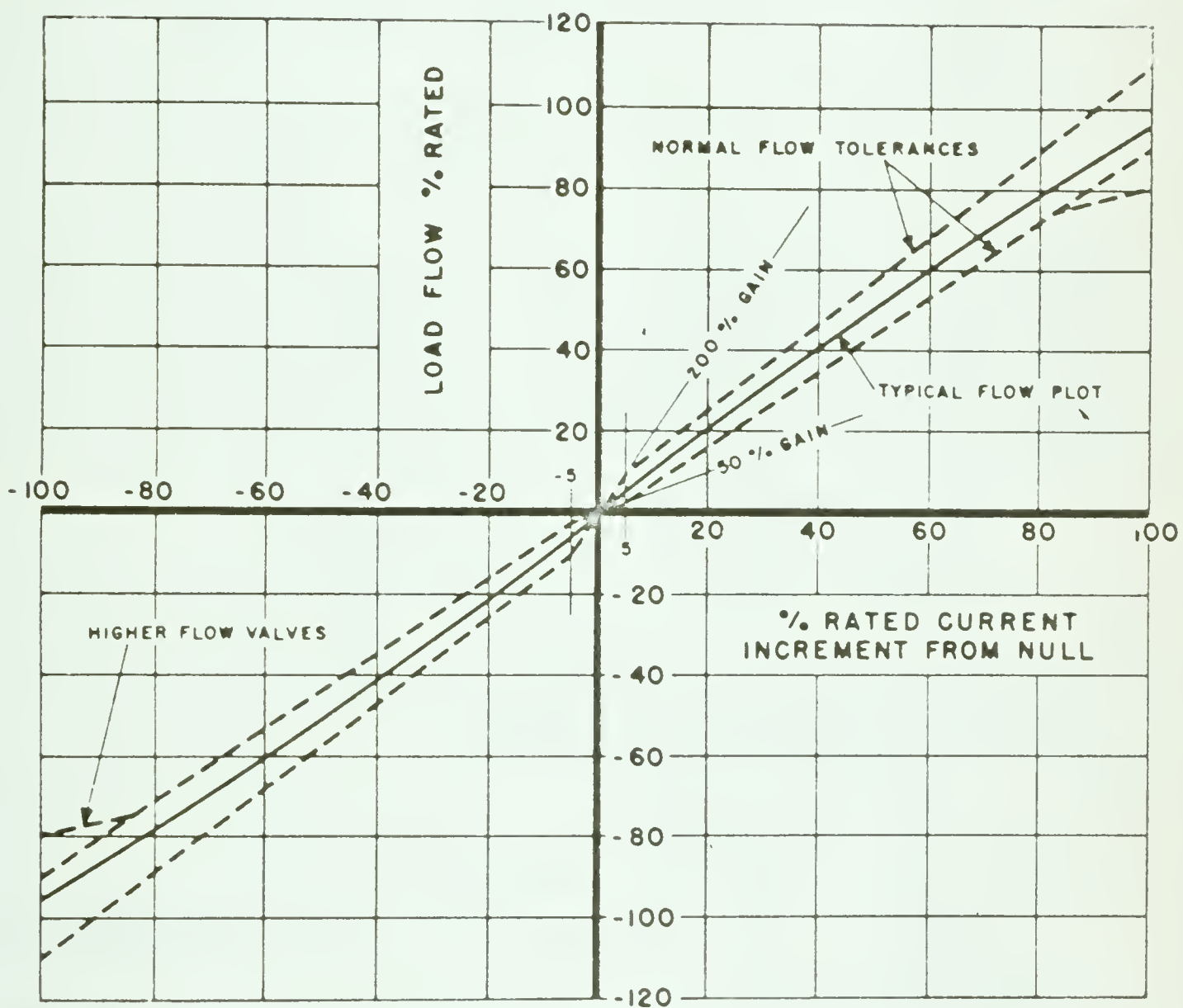


Figure 3  
Flow Gain Curve of Moog 71-101 Valve



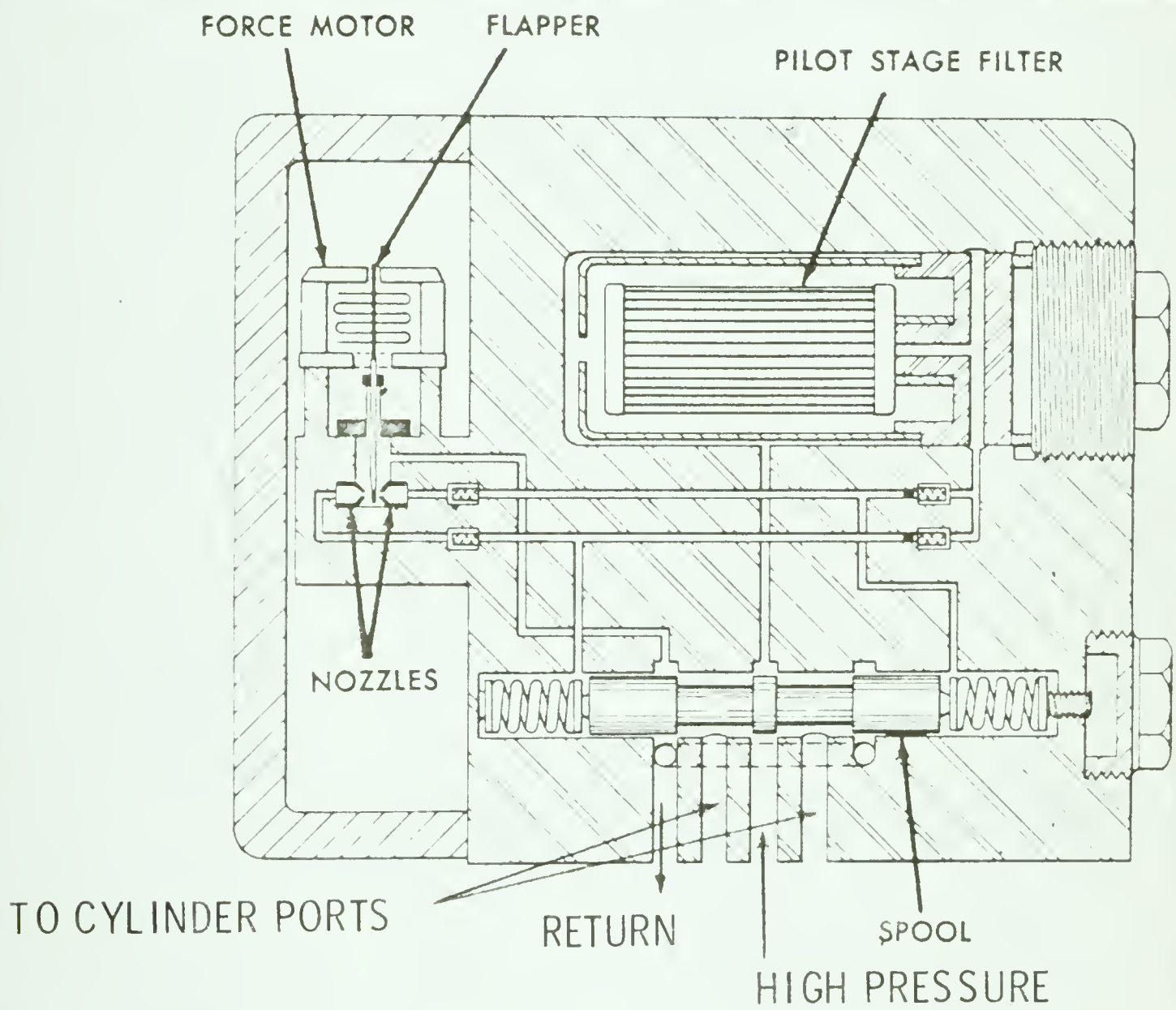


Figure 4

Schematic of Moog 71-101 Valve





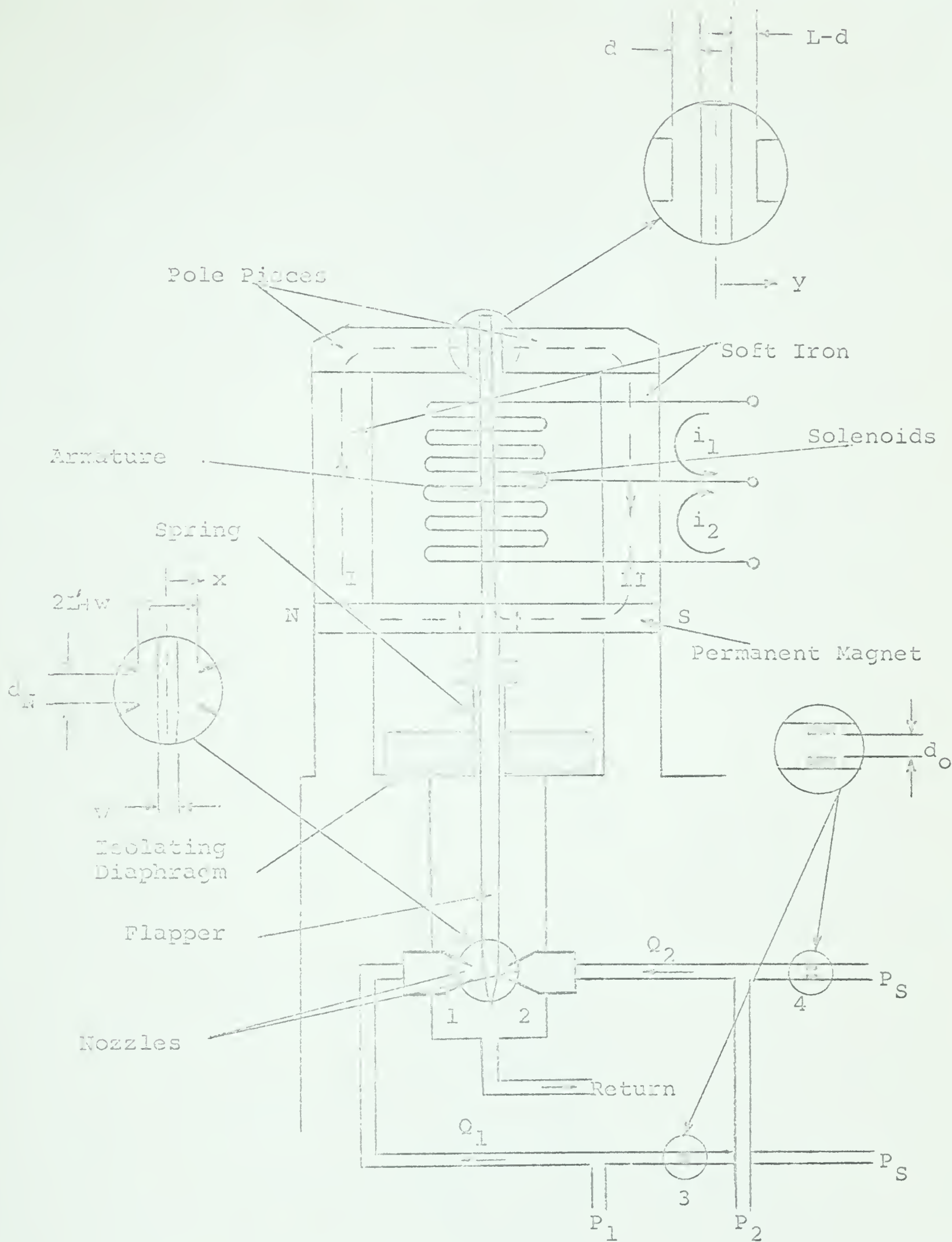


Figure 5

Detail of Torque Motor and Flapper Valve



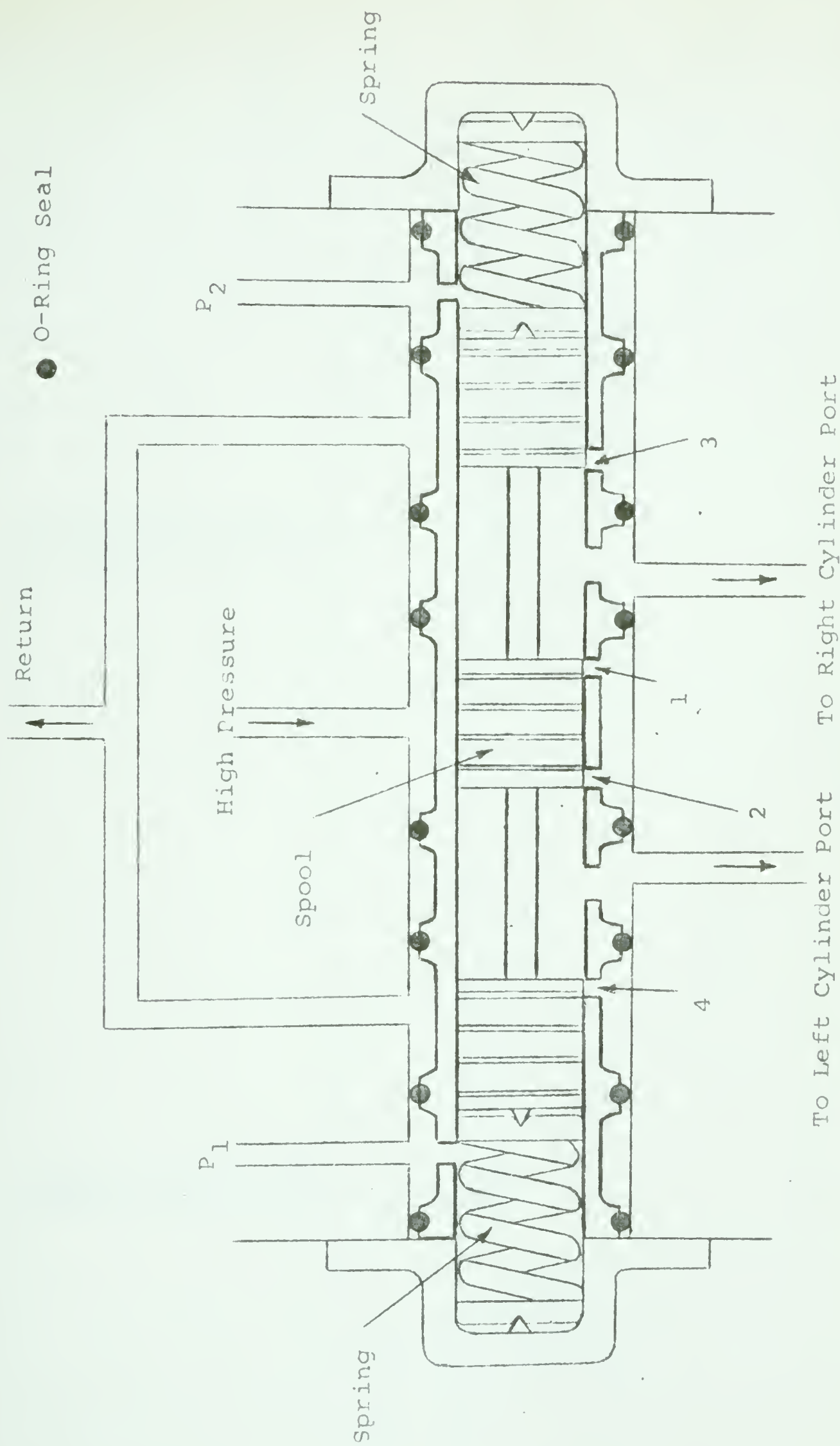


Figure 6

Detail of Spool Valve



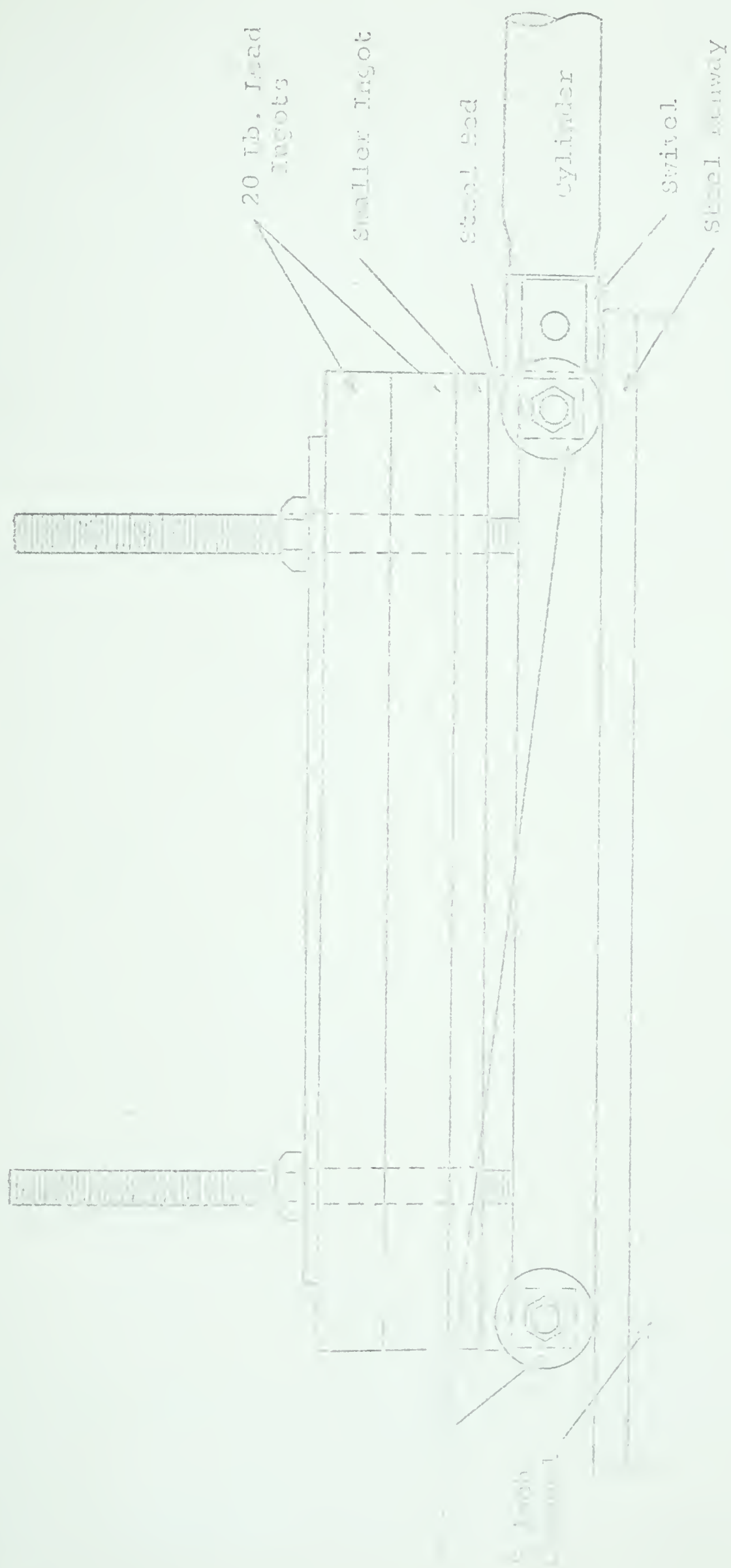


Figure 7

System Model





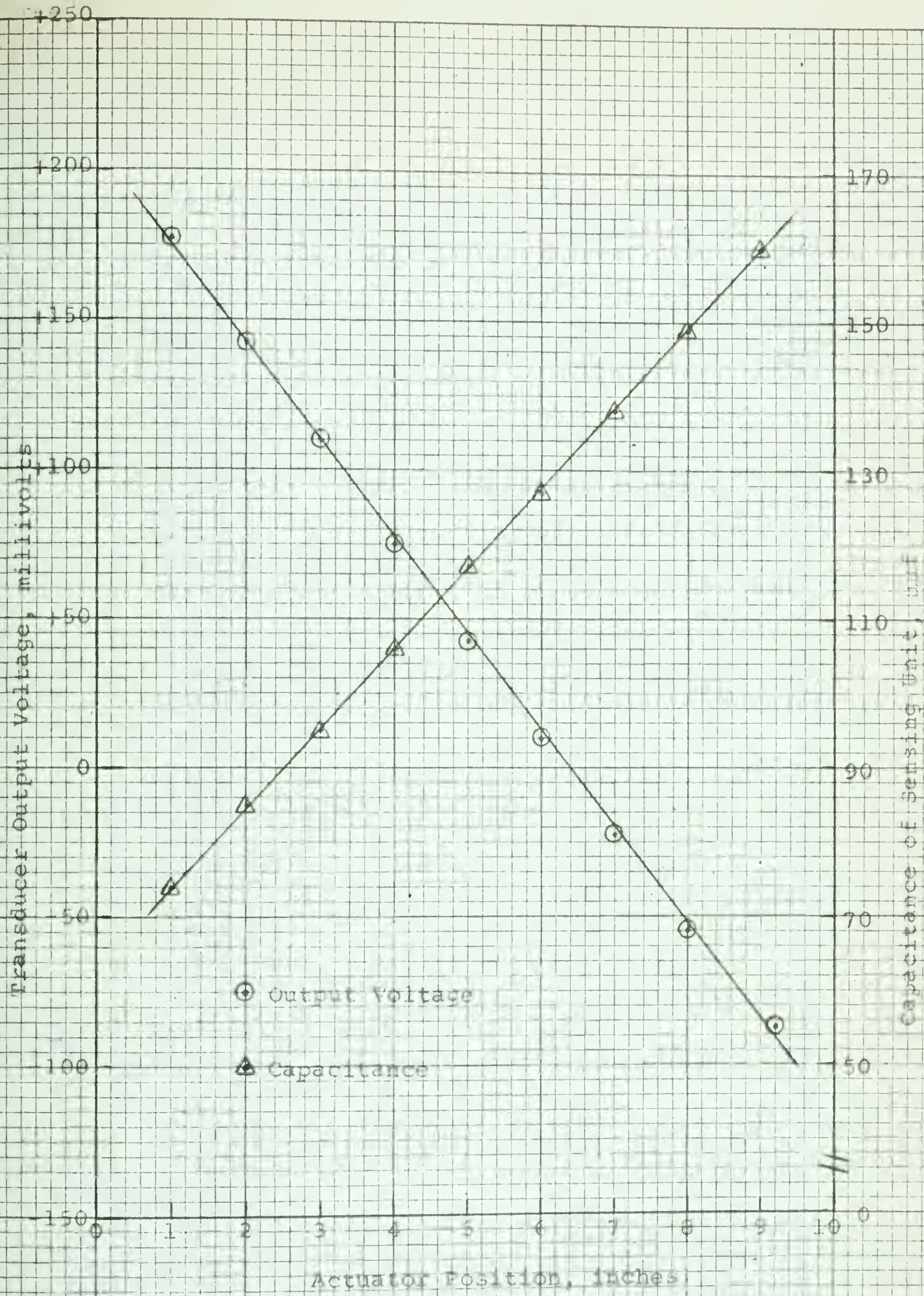


Figure 3

Capacitance and Control Characteristics of Position Transducer





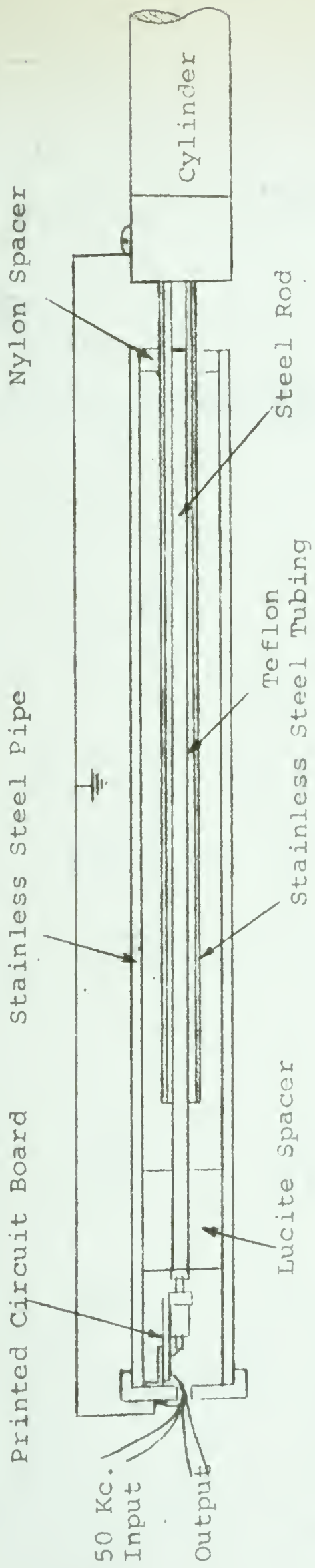


Figure 9a-Section of Position Sensing Element

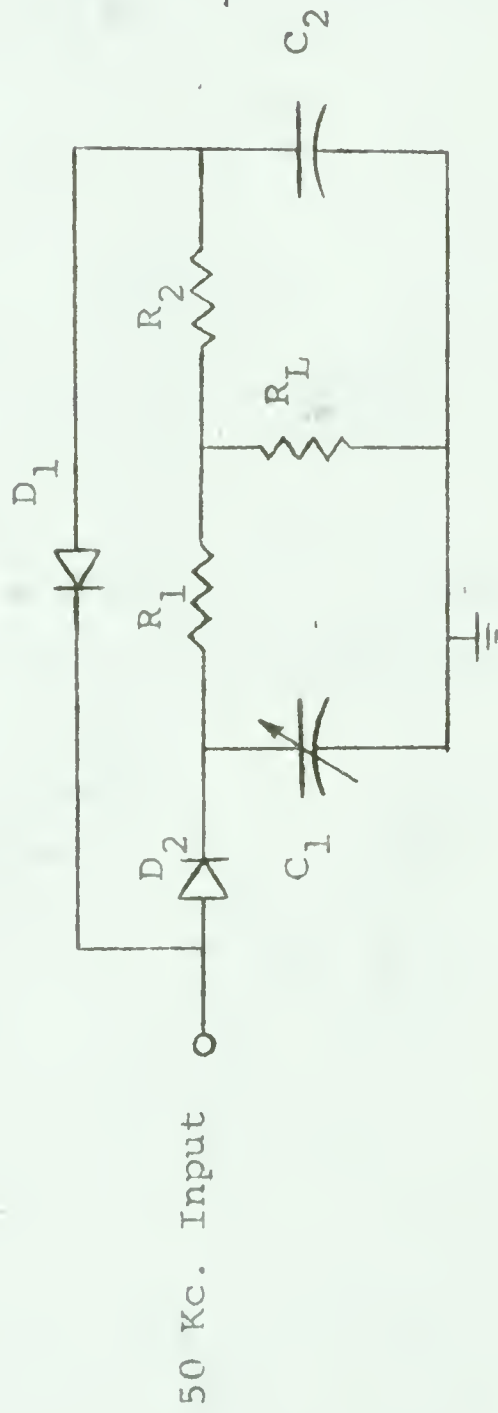


Figure 9b - Diode-Capacitor Bridge of Position Transducer

Figure 9



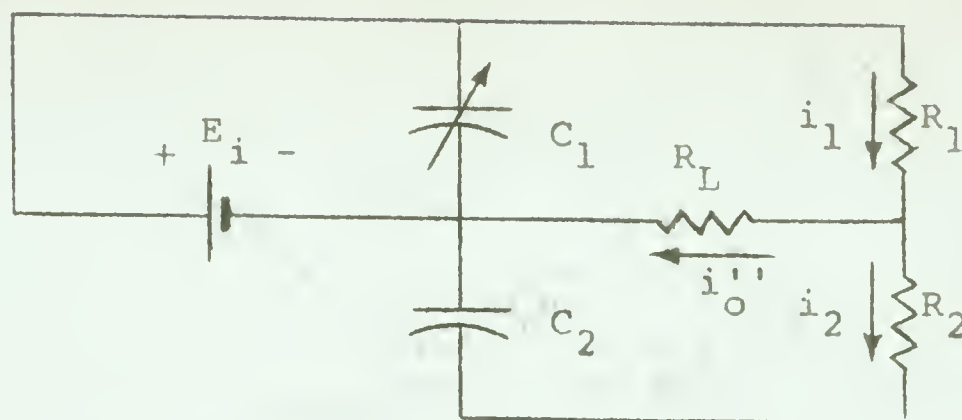


Figure 10a - Equivalent Circuit For Positive Half Cycle

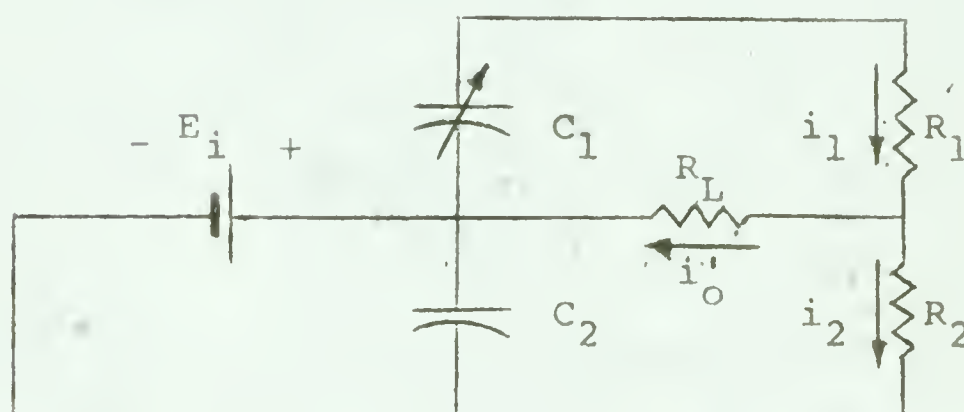


Figure 10b - Equivalent Circuit For Negative Half Cycle

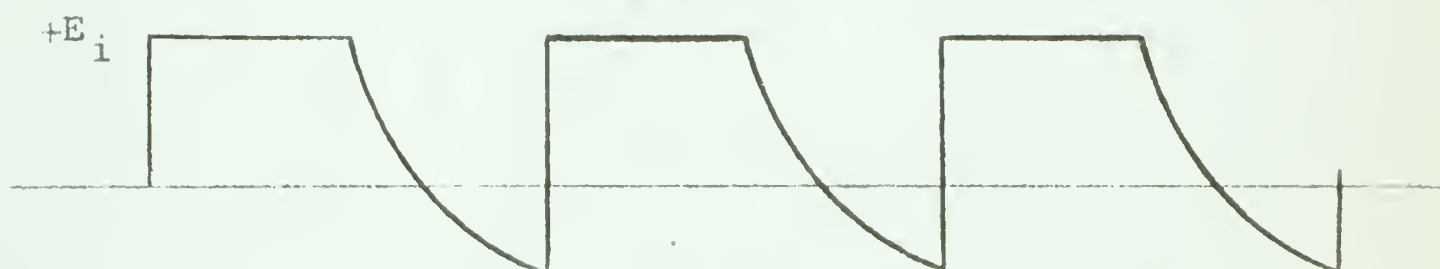


Figure 10c - Waveform For Capacitor  $C_1$

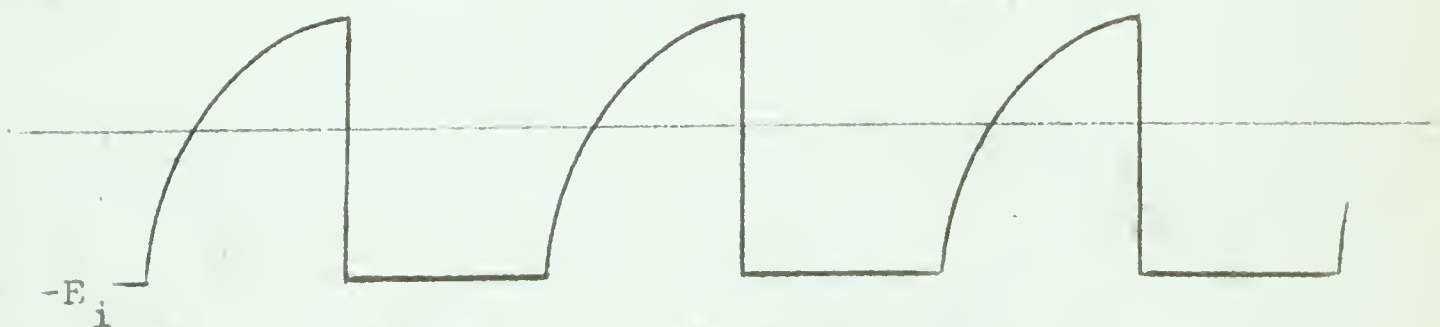


Figure 10d - Waveform For Capacitor  $C_2$





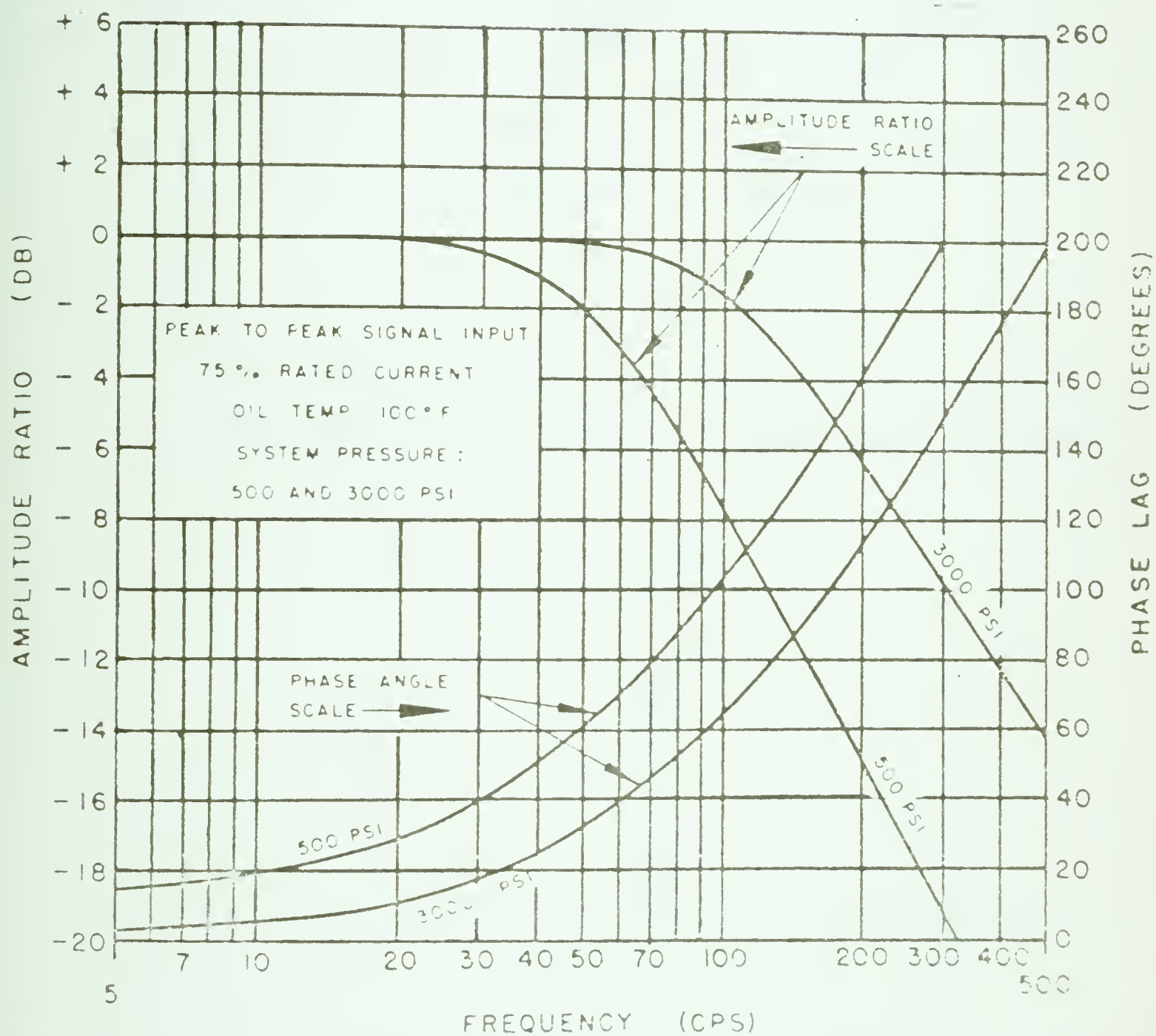


Figure 11

Frequency Response Curves for Moog 71-101 Valve



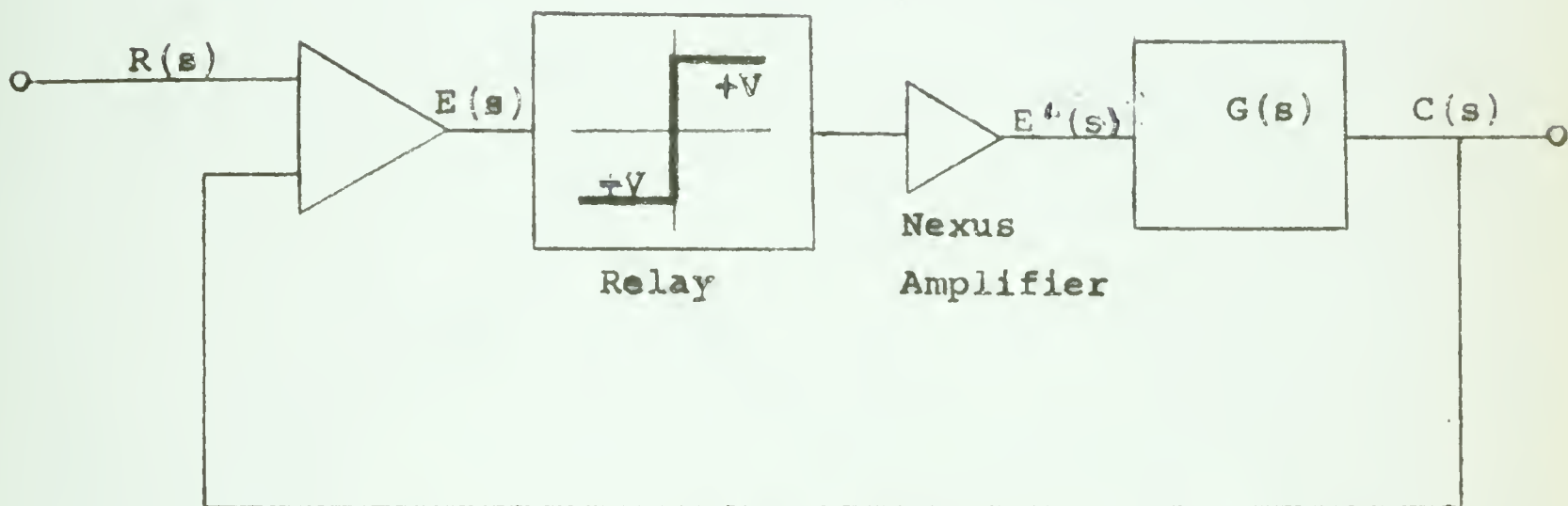


Figure 12a, Block Diagram of Relay-Controlled Servo

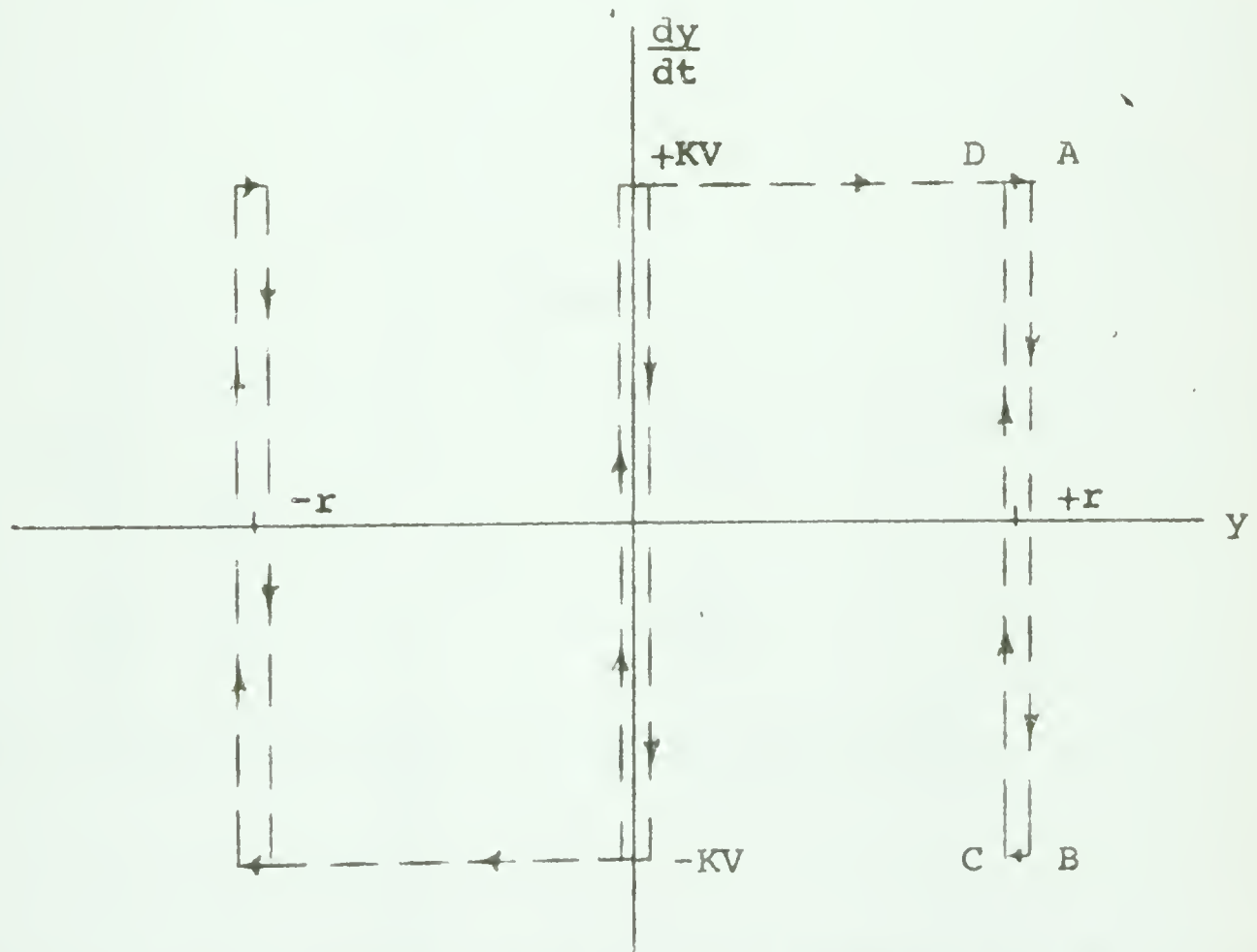


Figure 12b, Phase Plane Curve of Relay-Controlled Servo



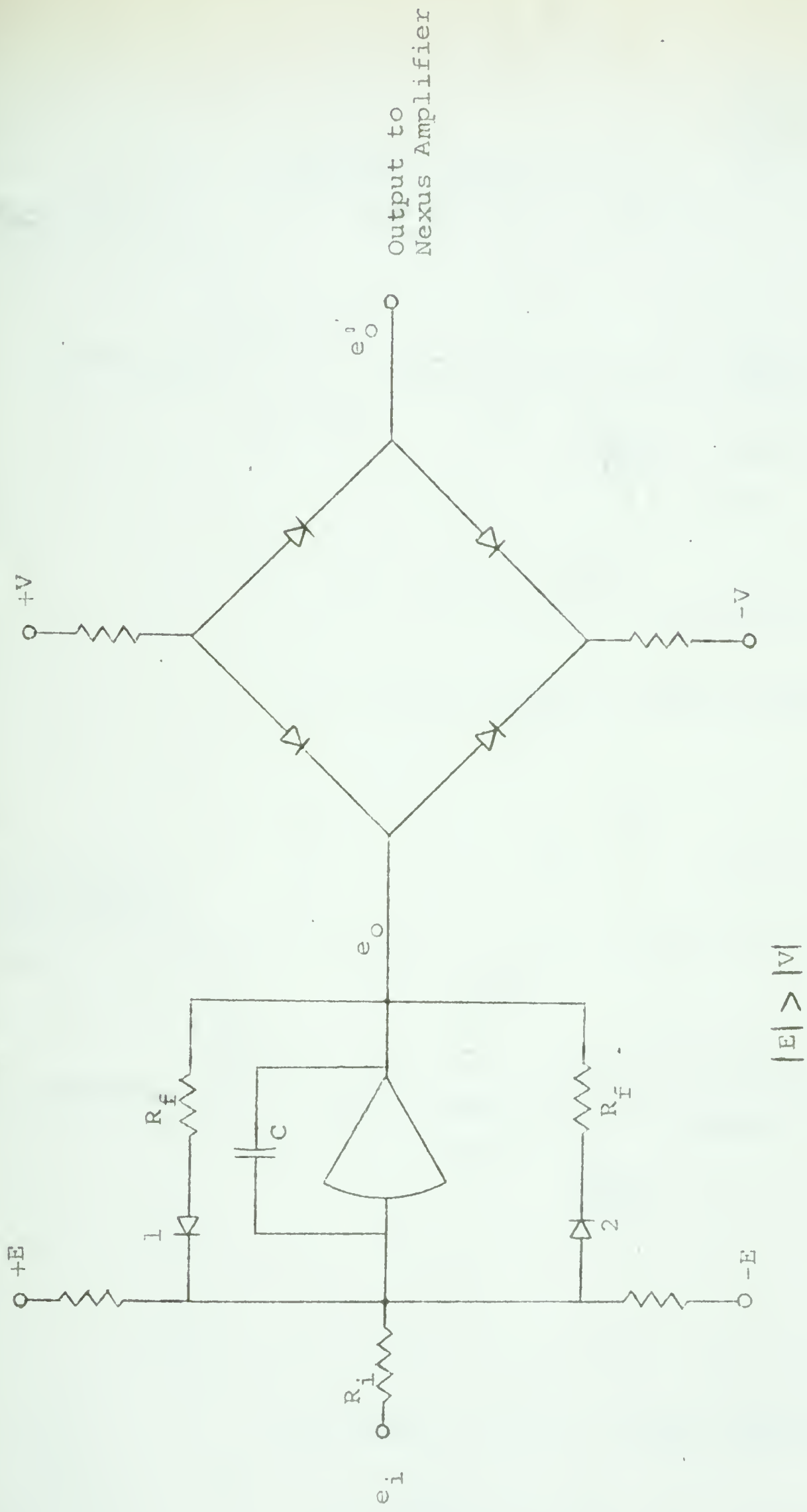
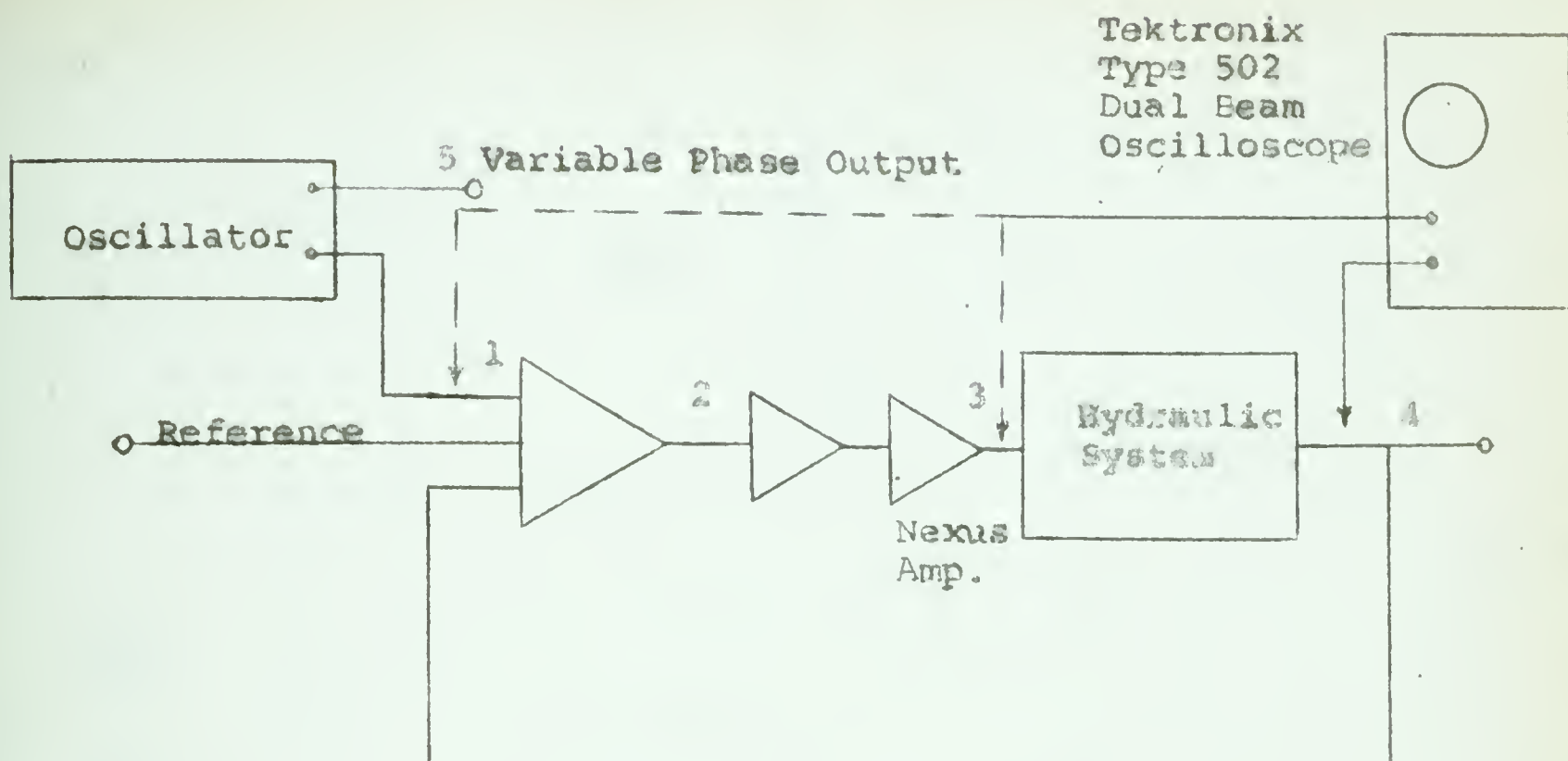


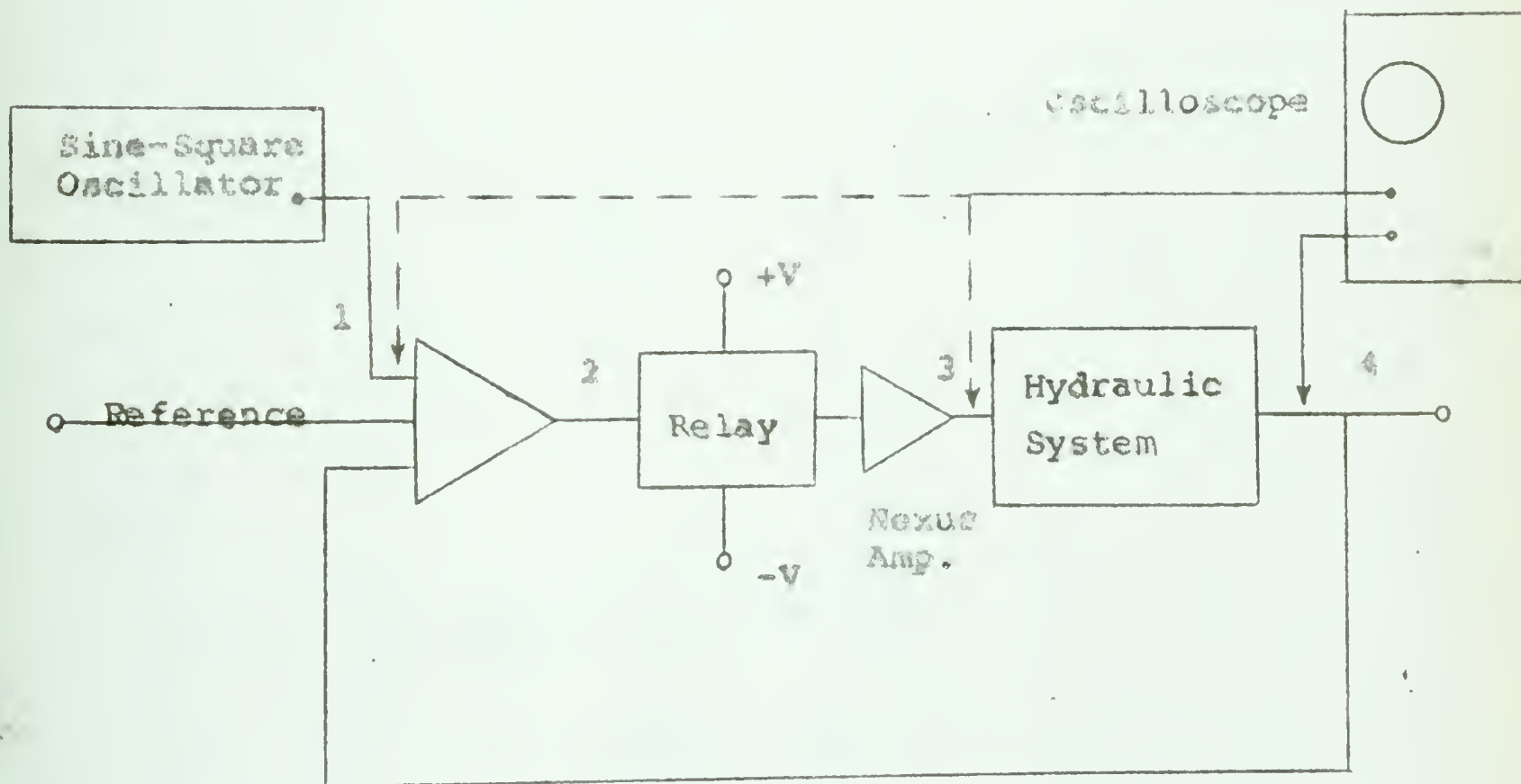
Figure 13







14a Testing circuit - Linear Operation



14b Testing Circuit - Relay Operation

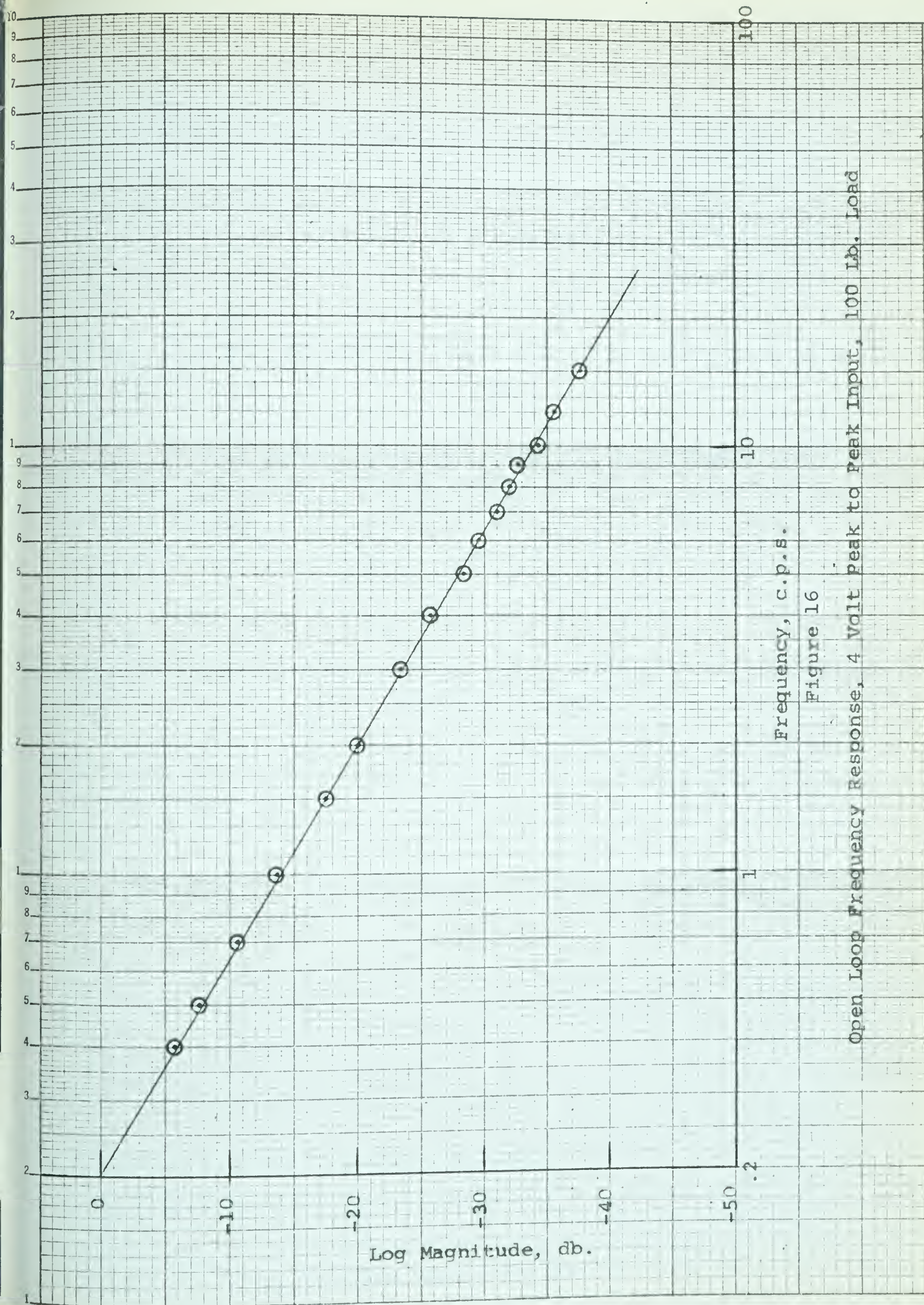
Figure 14  
Testing Circuits











Log Magnitude, db.

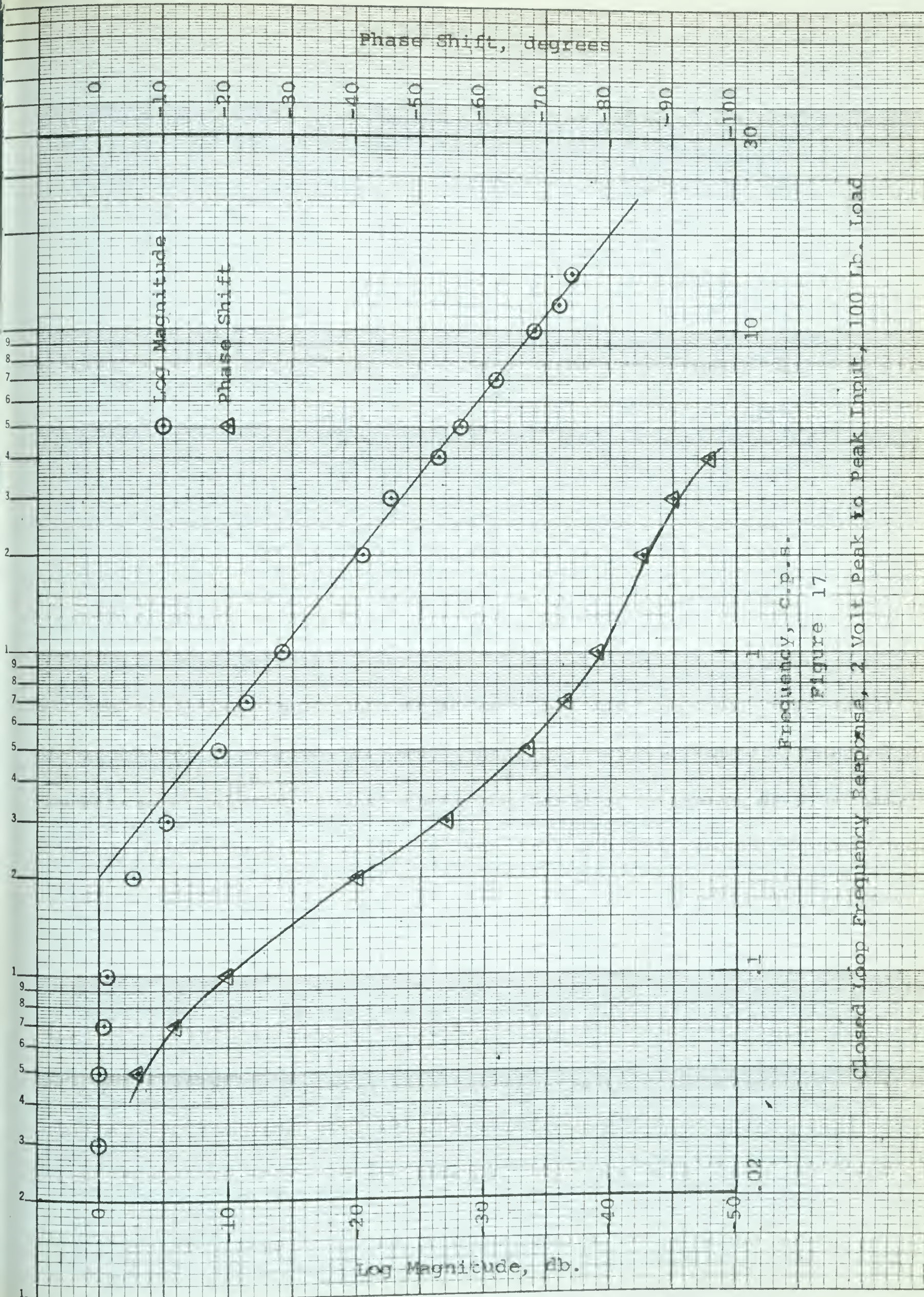
Frequency, c.p.s.

Figure 16

Open Loop Frequency Response, 4 Volt peak to peak Input, 100 lb. Load







Frequency, c.p.s.

Figure 17

Closed Loop Frequency Response, 2 Volt Peak to Peak Input, 100 lb. Load





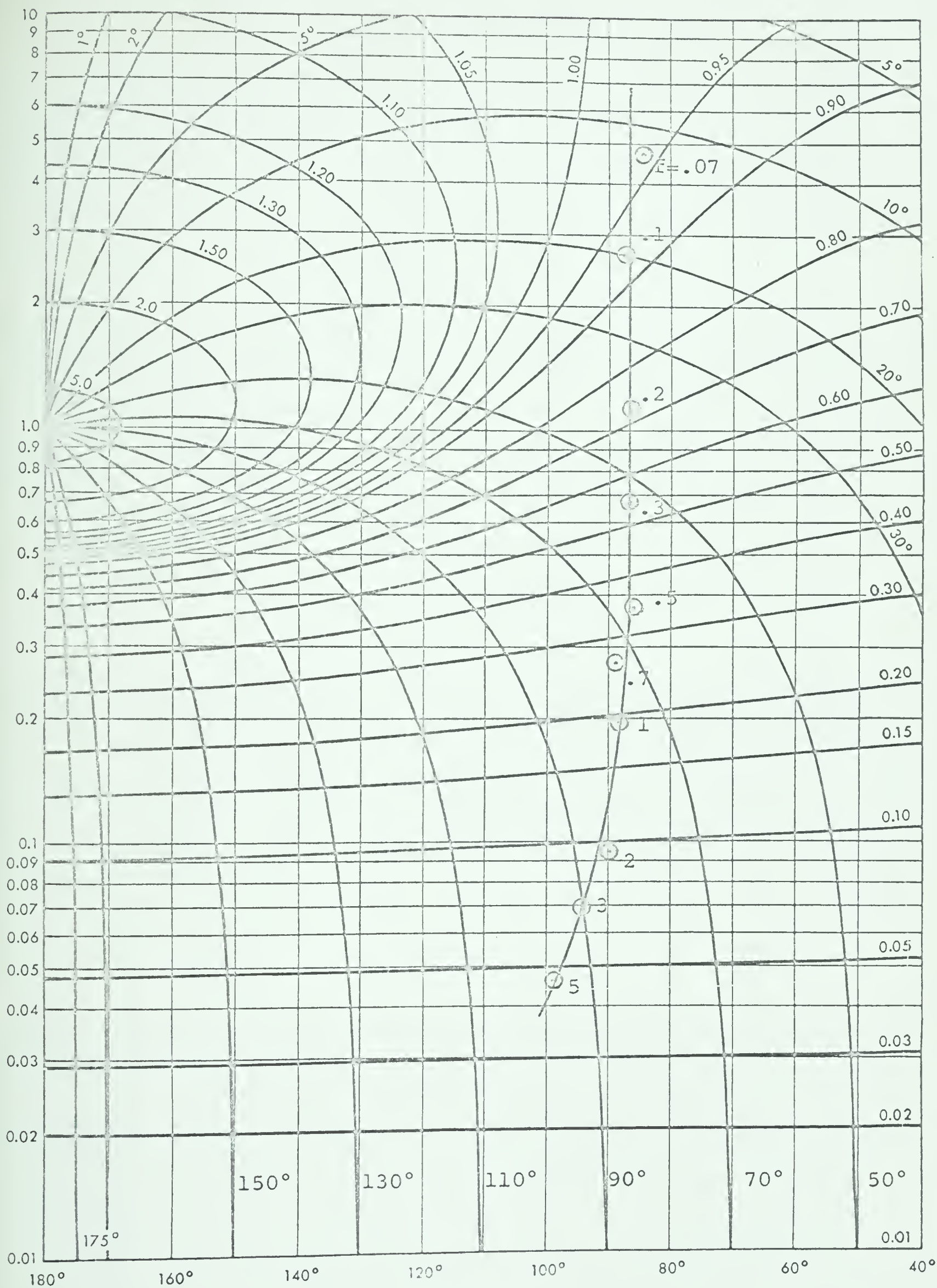
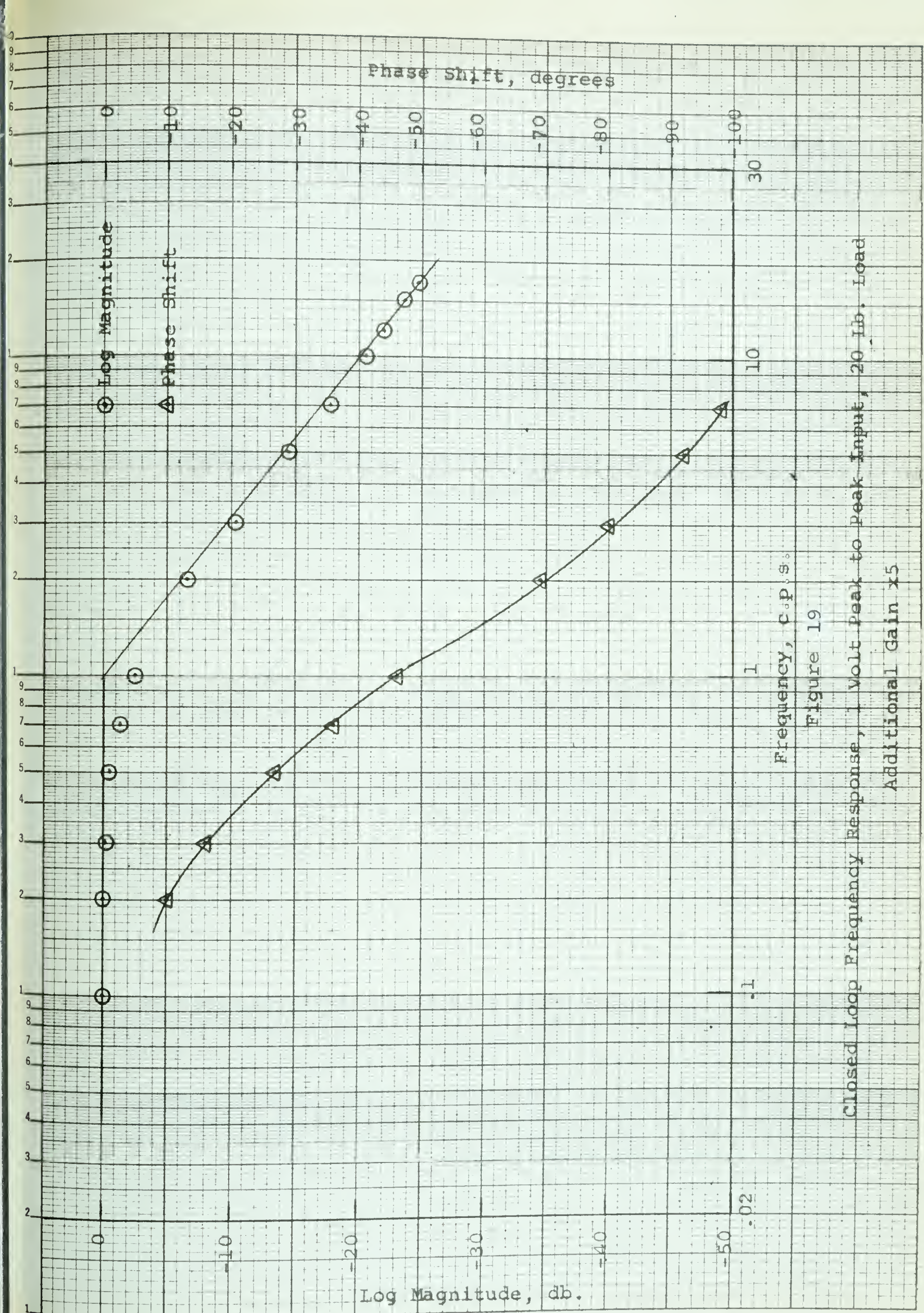


Figure 18  
 Nichols Diagram of Closed Loop Frequency Response  
 2 Volt Peak to Peak Input, 100 Lb. Load













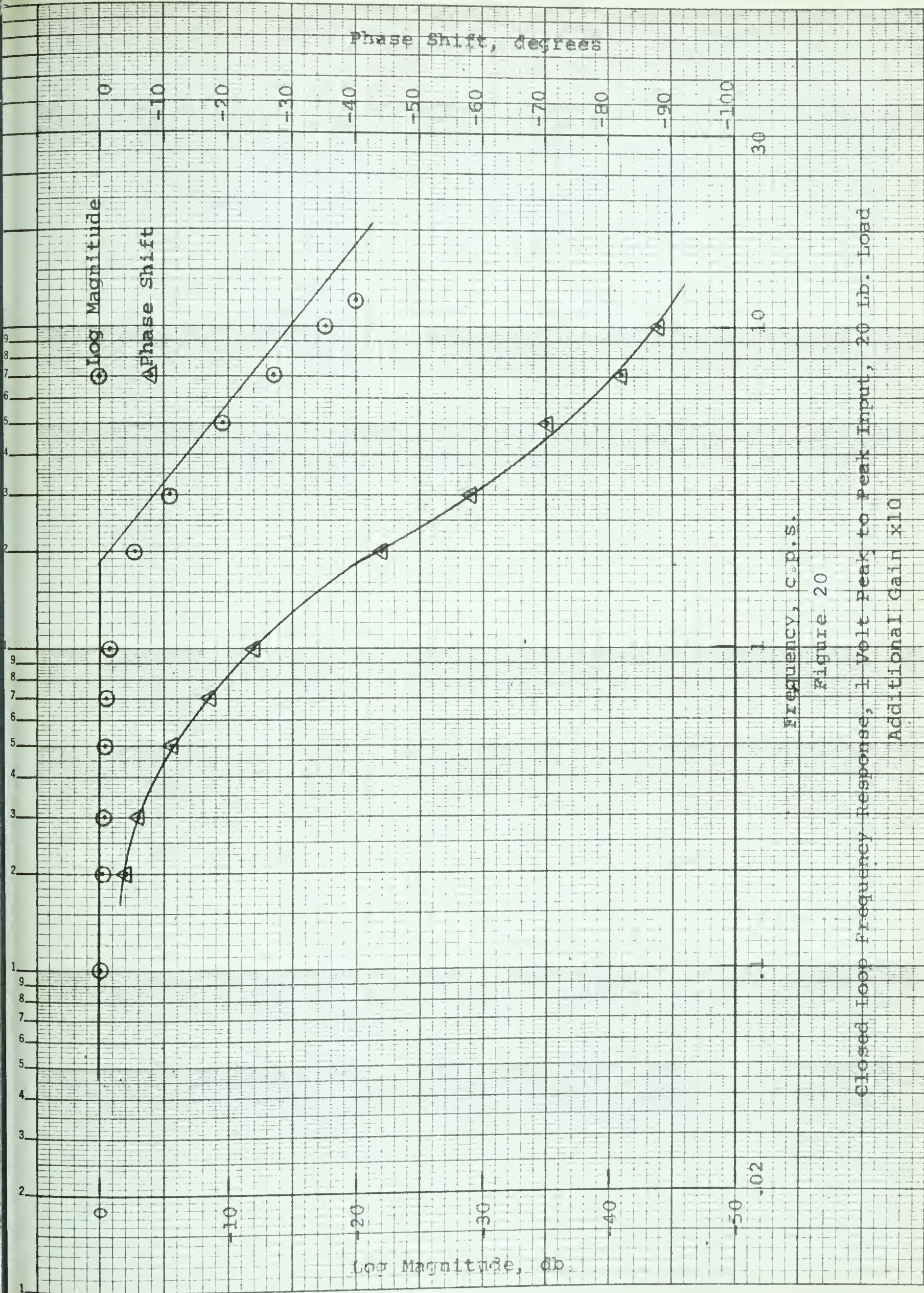


Figure 20

Closed Loop Frequency Response, 1 Volt Peak to Peak Input, 20 lb. Load  
Additional Gain x10





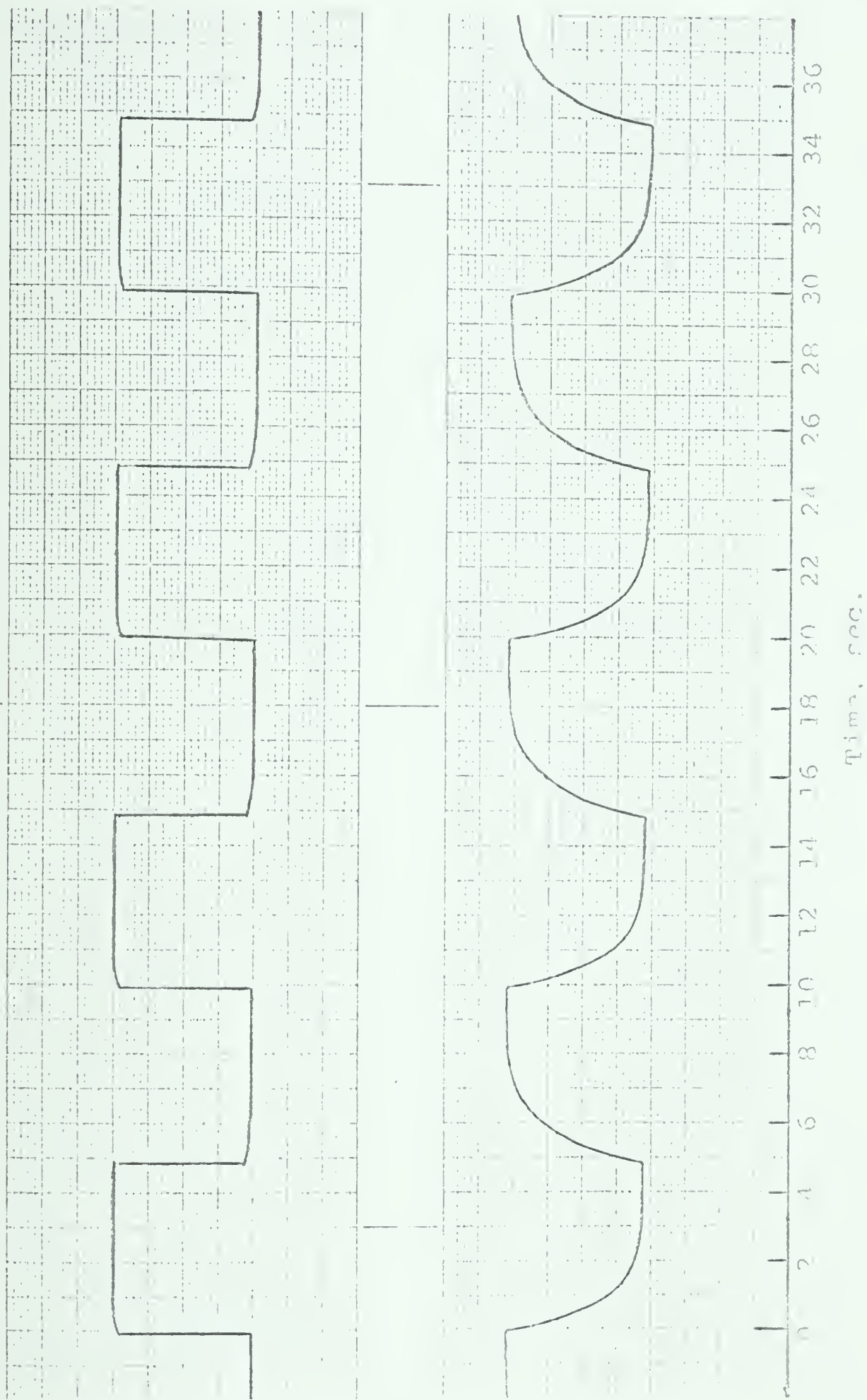
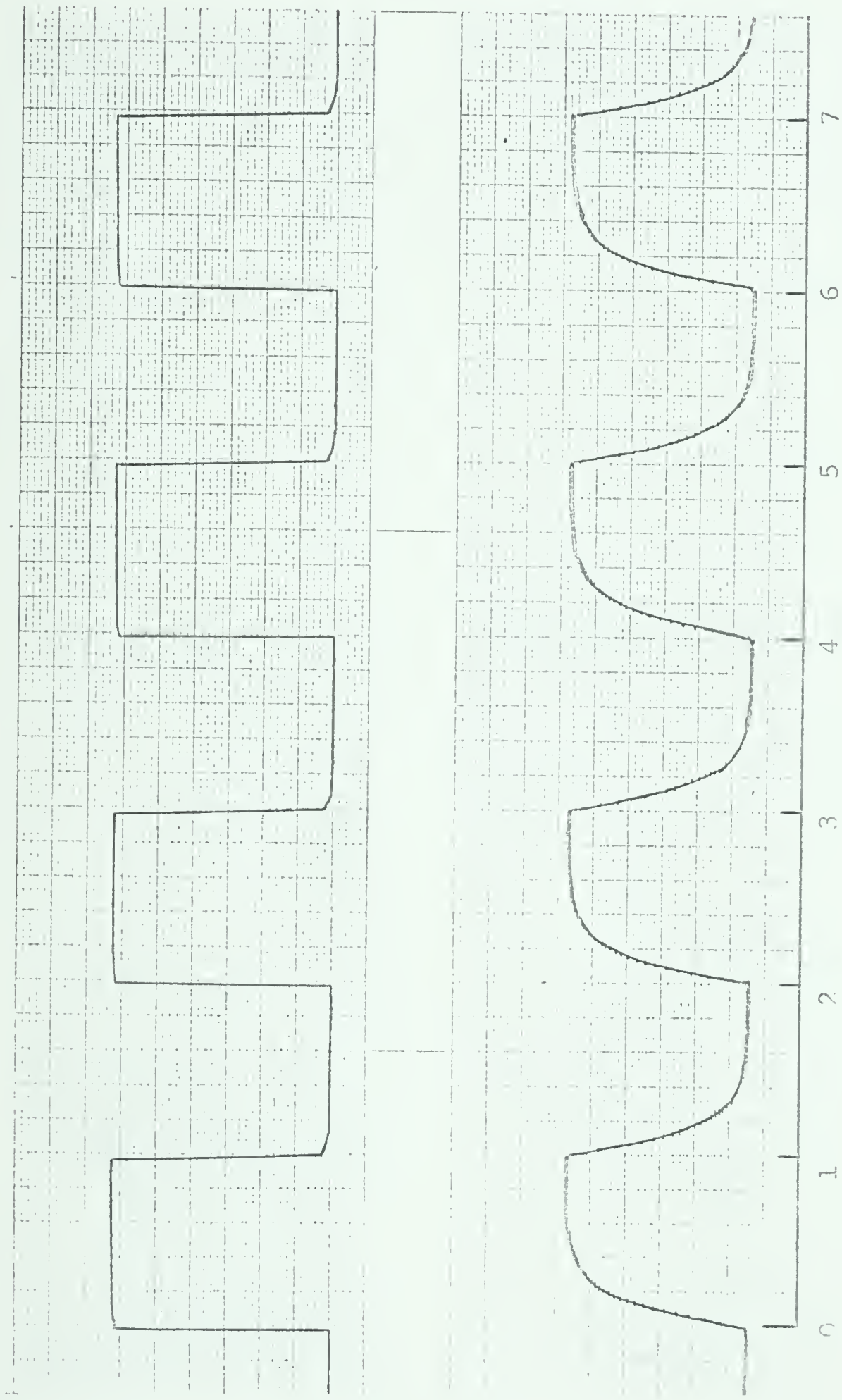


Figure 21

Frequency response - Linear system, 100 Hz. Load, 1 Volt Peak to Peak Input





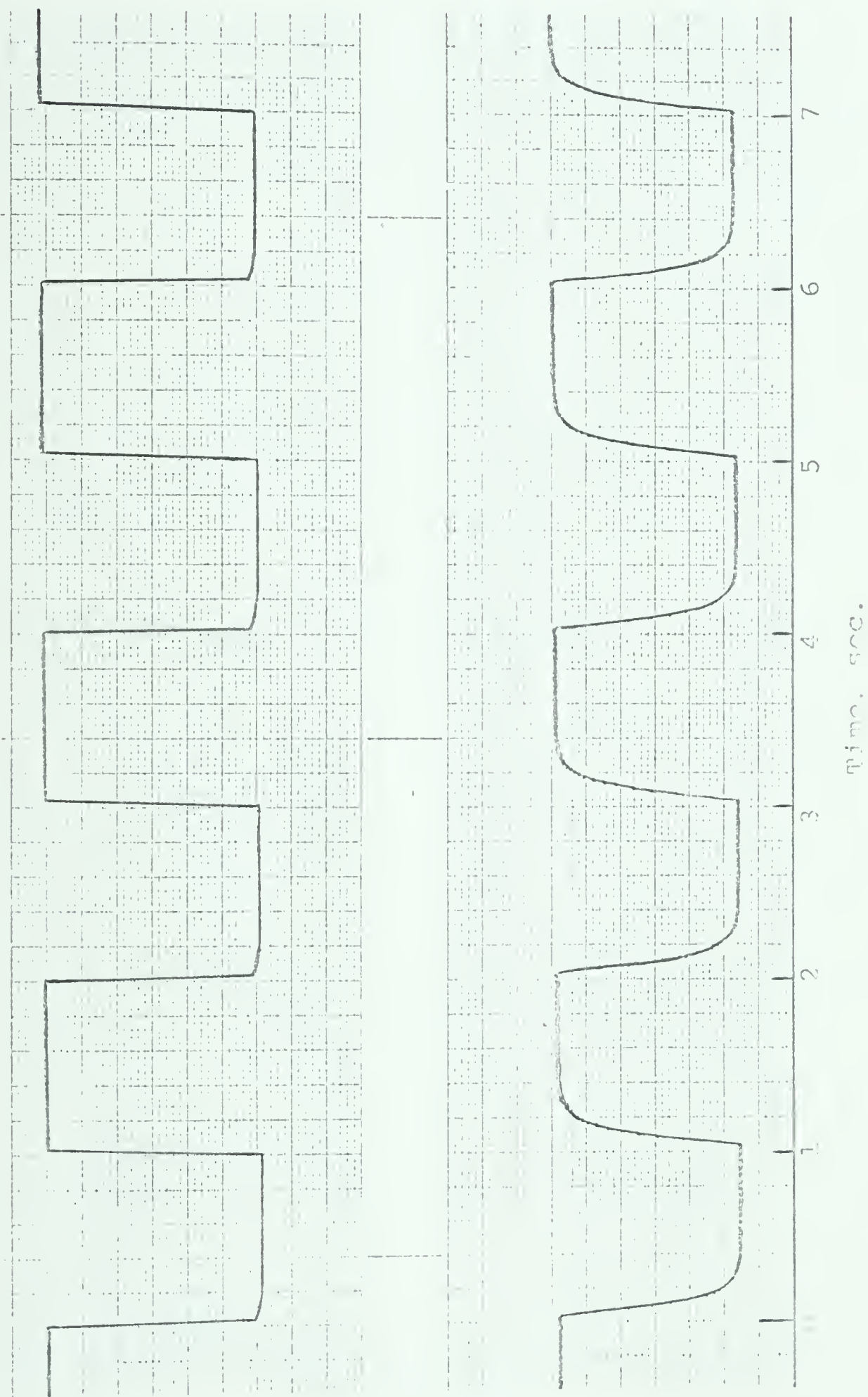
Time, sec.  
 Input trace - input  
 Output trace - output

Figure 22

Figure 22 shows the response of the system to a square wave input. The input signal is a square wave with a period of 1 second. The output signal is a smoothed version of the input, showing rounded transitions between high and low states, characteristic of a low-pass filter response. The system has a bandwidth of 5 Hz.







Upper trace - input

Lower trace - output

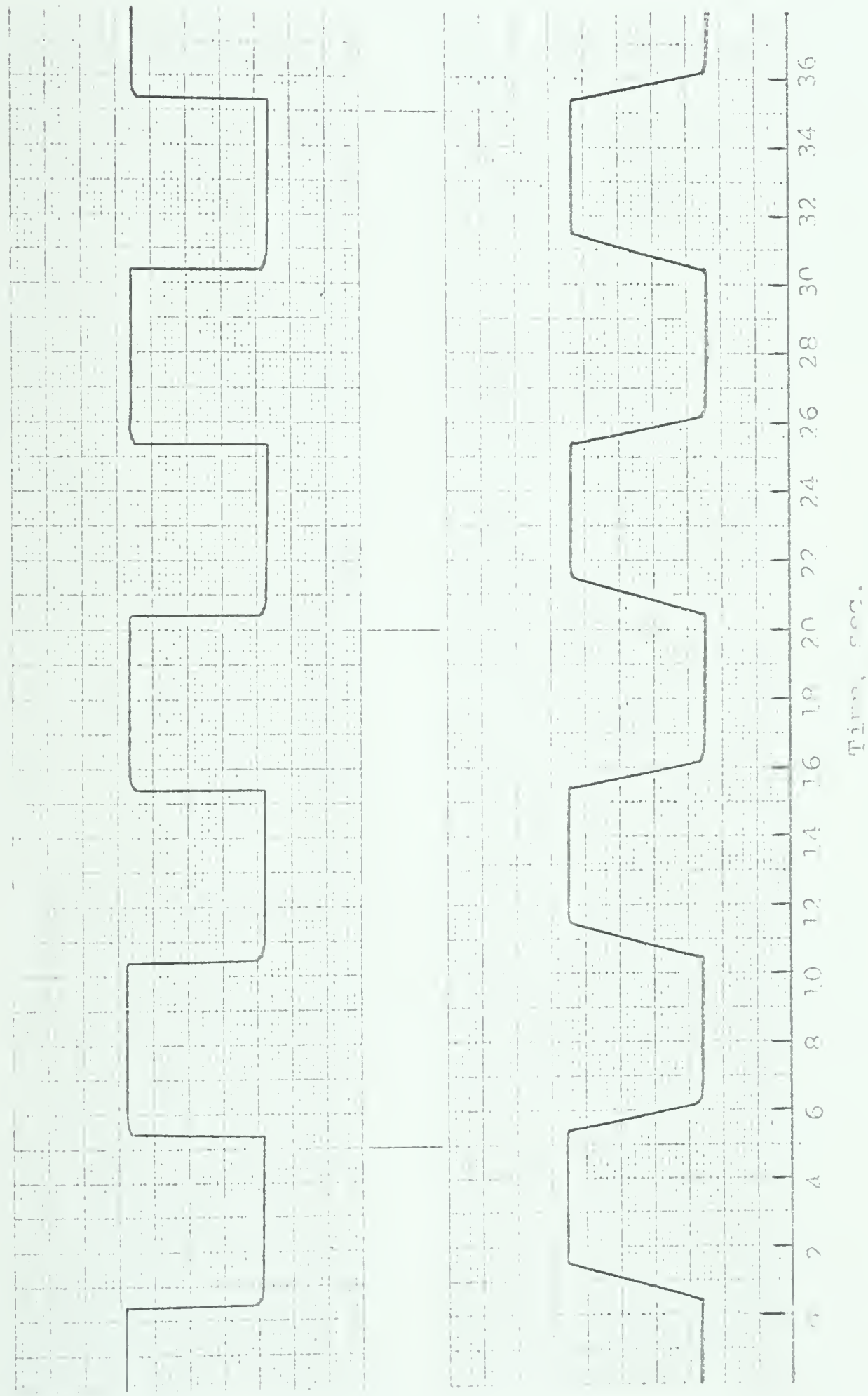
Figure 23

Transient response - linear system, 20 Hz, .5 volt peak to peak input  
additional gain 10









Input trace - input

Output trace - output

Figure 25

Timing diagram showing input and output waveforms. The input is a square wave with a period of 4 seconds and a peak-to-peak voltage of 1.0 volt. The output is a ramp wave that increases linearly from 0.0 V to 1.0 V over the 4-second high period of the input, and then decreases linearly back to 0.0 V over the 4-second low period of the input.







**B29830**